Math U575 Spring 09 HQuiz 1Prof. A. Iarrobino Name:_____

1. Prove that if a,b are elements of a group, then $a^2b^2 = (ab)^2 \iff ab = ba$.

2. Prove by mathematical induction that in a group, $\left(a^{-1}ba\right)^n = a^{-1}b^n a$, *for* $n \in \mathbf{N}$.

3. Let A be a point on a circle X, and G the group of geometric symmetries of X.
a. Find the stabilizer subgroup $Stab_G A$ of elements in G leaving A fixed.
b. Determine the orbit of A under G.
c. Use a,b to determine all the elements of G.

4. In the group D_6 of symmetries of the regular hexagon X with vertices A,B,C,D,E.F ordered counterclockwise, the flip f about the axis AD is given by f =(BF)(CE); and the flip g about the axis BE is given by (AC)(DF).

a. Determine the composition $g \circ f$.

b. Let τ = (ACE)(BDF). Identify τ as a geometric motion of X.

c. Determine $\tau^2 \circ f$

d. What is the order of D_6? Why?

e. Is D_6 Abelian? Why or why not?

5*. Make up and solve a problem in group theory, suitable for a HW problem next week.