Worksheet 3 Math U575 Spring 08 Prof. A. Iarrobino Name___________

1. Prove that in every column of the Cayley table of an arbitrary group, each element of the group appears exactly once.

a. State the cancellation law that is equivalent to no-element appears twice in the g column.

b. State the equation whose solution is equivalent to "h appears in the g column".

c. Prove each of a,b, from the definition of a group.

2. Show that the alternating group G= A_4 of even permutations of {a,b,c,d} is solvable (there is a sequence of normal subgroups whose successive quotients are Abelian), by showing that

i. The subset H={(ab}(cd), (ac)(bd),(ad)(bc),e} is a subgroup of G.

ii. H is a normal subgroup of G.

iii. The quotient G/H is isomorphic to Z_3.

iv. List the cosets of H in G.

(*Hint, this work is simplified if you consider G as the group of rotations of the tetrahedron with vertices a,b,c,d, and H as the isotropy group of the set P={{(ab)(cd)},{(ac)(bd)}{(ad)(bc)}} of pairs of opposite vertices.*

3. Consider the following groups G and subgroups H.
Determine the order of H, the left cosets of H in G, whether H is normal, and,if so, the quotient G/H is isomorphic to what group we have studied.

a. H=5Z in G=Z.

b. H=<5> in G=Z_{30}.

c. H=<(3,0)> in $Z_6 \times Z_4$.

d. H=<(1,2)> in S_3. *the permutations of {1,2,3}.*

e. H= $A_4 \subset S_4$, *the permutations of {a,b,c,d}*

4a. Prove that the map $\theta : D_4 \to Z_2$, θ *(g)=0 if g is a rotation, and*
θ *(g)=1 if g is a flip*
is a homomophism of groups, with kernel isomorphic to Z_4.

4b. Show that there is no surjective homomorphism π from $D_4 \to Z_4$.. *(Hint, show that the kernel of π must have order 2; then show that there is no normal subgroup of* D_4 *of order 2: that is no subgroup (f,e), f a flip, is normal.*