

Assignment 1 for MTH 131: Fall 2006

Due date: Wednesday September 13.

Reading: Sections 1.1, 1.2 in Logan; also review one-variable calculus from any undergraduate text.

1). Limits

(i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{1 - x}$ (ii) $\lim_{x \rightarrow 0} \cos(x)$ (iii) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ (iv) $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^2}$

2). Continuity

a) Show that the function $f(x) = 1 + x^2$ is continuous at $x = 0$ as follows: for any fixed positive number a , find a positive number b so that the interval $[-b, b]$ around $x = 0$ gets mapped inside the interval $J = [1 - a, 1 + a]$ around $y = 1$ by the function f . If you can do this for any value of a (no matter how small) then you are done!

b) Try to repeat the argument above for the function $g(x)$ which is equal to $f(x)$ on the right side (that is for $x > 0$) but is identically zero on the left side (that is for $x < 0$). Can you choose a value for $g(0)$ which makes it continuous at $x = 0$? If not, why?

c) Make up a definition of 'one-sided' continuity (from the right side) for a function $f(x)$ at the point $x = 0$ (use ϵ 's and δ 's). By applying your definition to the function $g(x)$ in (b) above, show that you can choose a value for $g(0)$ so that $g(x)$ is continuous from the right side at $x = 0$.

3). Derivatives

Compute derivatives of these functions:

(i) $(x^2 + 3)^4$ (ii) $\exp(x^2 + 5x)$ (iii) $\cos(x^2 + 1)$ (iv) $(x - 1)^{-5}$
(v) $\log(\cos(x^{1/2} + 4))$

4). ODE's I

a) Show that the function $y = \sin(1/x - C)$ is a solution of the following ODE for any number C :

$$x^2 \frac{dy}{dx} + \sqrt{1 - y^2} = 0$$

b) Note that the solution in (a) is not continuous at $x = 0$ for any value C . Question: can you find a different solution which is continuous at $x = 0$?

5). ODE's II Verify that the second order ODE

$$\frac{d^2y}{dx^2} + y = 1$$

has the general solution

$$y = 1 + A \sin(x) + B \cos(x)$$

where A, B are any numbers.

a) Find the values of A, B which solve this equation with boundary values $y(0) = 0$ and $y(\frac{\pi}{2}) = 0$.

b) Try the same for boundary values $y(0) = 0$ and $y(\pi) = 0$. What do you find?

c) Try the same for boundary values $y(0) = 0$ and $y(\pi) = 2$. What do you find?

6). ODE's III

Logan p.11: Problems #4,#7.