

## Assignment 6 for MTH G131: Fall 2006

**Due date:** Wednesday October 18.

**Reading:** Logan, Chapter 5; undergraduate linear algebra text.

**1). [Matrices]**

Logan p.177: #5, #16, #18.

**2). [Matrices]**

Logan p.196: #1 (C only), #2, #4 (a), (b), (f), #9, #10.

**3). [Modeling: projectile with air resistance]**

A projectile is launched at an angle  $\theta$  to the horizontal, with initial speed  $V$ . The motion occurs in the  $xy$  plane, starting at the origin, so the initial conditions are

$$x(0) = 0, \quad y(0) = 0, \quad \dot{x}(0) = V \cos(\theta), \quad \dot{y}(0) = V \sin(\theta)$$

Assume that air resistance acts in the direction opposite to the velocity, with magnitude  $k|v|$  where  $v$  is the velocity. Then the equations of motion are

$$m\ddot{x} = -k\dot{x}, \quad m\ddot{y} = -k\dot{y} - g$$

where  $m$  is mass of the projectile and  $g$  is the acceleration due to gravity, and 'dots' denote derivatives with respect to  $t$ .

a) Let  $v_1 = \dot{x}$  and  $v_2 = \dot{y}$  be the components of the velocity. Rewrite the equations of motion in terms of  $v_1$  and  $v_2$ , and solve them for  $v_1$  and  $v_2$  as functions of  $t$ , using the initial conditions for the velocity.

b) The maximum height is reached when  $\dot{y} = 0$ . Solve the equation  $v_2(t) = 0$  from part (a) to find the time  $t_1$  when the projectile reaches its maximum height.

c) Replace  $v_1$  by  $\dot{x}$  and  $v_2$  by  $\dot{y}$  in the solutions from part (a), and solve these equations for  $x$  and  $y$  as functions of  $t$  (using the initial conditions  $x(0) = y(0) = 0$ ).

*d)* Solve the equation  $x = x(t)$  from part (c) to find  $t$  as a function of  $x$ . Substitute this into  $y = y(t)$  to find the equation of the trajectory in the form  $y = f(x)$ .

*e)* In the limit  $k \rightarrow 0$  your solution in part (d) should be a parabola, that is it should have the form  $y = -cx^2 + dx$  for some constants  $c$  and  $d$ . Verify that this is true.