

Assignment 6 for MTH G131: Fall 2006

Due date: Wednesday October 18.

Reading: Logan, Chapter 5; undergraduate linear algebra text.

1). [Matrices]

Logan p.177: #5, #16, #18.

2). [Matrices]

Logan p.196: #1 (C only), #2, #4 (a), (b), (f), #9, #10.

3). [Modeling: projectile with air resistance]

A projectile is launched at an angle θ to the horizontal, with initial speed V . The motion occurs in the xy plane, starting at the origin, so the initial conditions are

$$x(0) = 0, \quad y(0) = 0, \quad \dot{x}(0) = V \cos(\theta), \quad \dot{y}(0) = V \sin(\theta)$$

Assume that air resistance acts in the direction opposite to the velocity, with magnitude $k|v|$ where v is the velocity. Then the equations of motion are

$$m\ddot{x} = -k\dot{x}, \quad m\ddot{y} = -k\dot{y} - g$$

where m is mass of the projectile and g is the acceleration due to gravity, and 'dots' denote derivatives with respect to t .

a) Let $v_1 = \dot{x}$ and $v_2 = \dot{y}$ be the components of the velocity. Rewrite the equations of motion in terms of v_1 and v_2 , and solve them for v_1 and v_2 as functions of t , using the initial conditions for the velocity.

b) The maximum height is reached when $\dot{y} = 0$. Solve the equation $v_2(t) = 0$ from part (a) to find the time t_1 when the projectile reaches its maximum height.

c) Replace v_1 by \dot{x} and v_2 by \dot{y} in the solutions from part (a), and solve these equations for x and y as functions of t (using the initial conditions $x(0) = y(0) = 0$).

d) Solve the equation $x = x(t)$ from part (c) to find t as a function of x . Substitute this into $y = y(t)$ to find the equation of the trajectory in the form $y = f(x)$.

e) In the limit $k \rightarrow 0$ your solution in part (d) should be a parabola, that is it should have the form $y = -cx^2 + dx$ for some constants c and d . Verify that this is true.