

## Assignment 7 for MTH G131: Fall 2006

**Due date:** Wednesday October 25.

**Reading:** Logan, Chapter 5, and Section 6.3; Matlab manual.

### 1). [Linear systems]

Each of the following linear systems has a critical point at the origin. In each case determine whether the critical point is stable, asymptotically stable, or unstable.

$$\mathbf{a)} \quad \frac{dx}{dt} = 3x - 2y, \quad \frac{dy}{dt} = 2x - 2y$$

$$\mathbf{b)} \quad \frac{dx}{dt} = x - 4y, \quad \frac{dy}{dt} = 4x - 7y$$

$$\mathbf{c)} \quad \frac{dx}{dt} = 2x - 5y, \quad \frac{dy}{dt} = x - 2y$$

$$\mathbf{d)} \quad \frac{dx}{dt} = 3x - 2y, \quad \frac{dy}{dt} = 4x - y$$

## 2). [Almost linear systems]

Each of the following systems has a critical point at the origin. In each case show that the system is almost linear (in the sense defined in class), and if possible determine whether the critical point is asymptotically stable or unstable.

$$\text{a)} \quad \frac{dx}{dt} = x - y + xy, \quad \frac{dy}{dt} = 3x - 2y - xy$$

$$\text{b)} \quad \frac{dx}{dt} = 2x + y + xy^3, \quad \frac{dy}{dt} = x - 2y - xy$$

$$\text{c)} \quad \frac{dx}{dt} = x + 2x^2 - y^2, \quad \frac{dy}{dt} = x - 2y + x^3$$

$$\text{d)} \quad \frac{dx}{dt} = 1 + y - e^{-x}, \quad \frac{dy}{dt} = y - \sin x$$

## 3). [Computer Lab 1: van der Pol equation]

The van der Pol equation  $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$  can be written as the two-dimensional system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x + \mu(1 - x^2)y$$

This computer lab allows you to select a value for the parameter  $\mu$ , and then compute the solution of this system for any initial values  $x(0), y(0)$  over any time interval  $[0, T]$ . The output has three parts: first, a plot of  $x(t)$  against  $t$  over the interval  $[0, T]$ ; second, a plot of  $x(t)$  against  $y(t) = \dot{x}(t)$  over the interval  $[0, T]$ ; third, a list of the values of  $t$  where  $x(t) = 0$  (these points are also marked on the first plot). The lab is available in the form of two text files (named ‘file 1’ and ‘file 2’) which are posted on the course webpage. You should copy these files and paste them into m-files when you run Matlab. The first one must be saved as “vdpol” and the second as

“vdpol2” (the editor will append .m to these names). To run the program type “vdpol2” in the Command window of Matlab.

**a)** For  $\mu < 0$  the system has an asymptotically stable critical point at the origin. Produce a nice picture that shows the solution converging to the origin for this case (for  $\mu$  close to zero you get a nice long spiral).

**b)** For  $\mu > 0$  the origin is unstable. However there is a stable periodic solution which attracts all the orbits. Produce a nice picture that shows the solution moving away from the origin and converging onto the periodic solution.

**c)** The third piece of output described above (the list of values of  $t$  where  $x(t) = 0$ ) allows you to find the period of the stable periodic solution from part **(b)**. Investigate how this period changes as  $\mu$  increases from zero to larger values. Calculate the period for enough values of  $\mu$  so that you can sketch a graph of period against  $\mu$ , and answer questions like: does the period increase or decrease? is the graph convex, concave or neither? does it converge to some simple behavior as  $\mu$  becomes large?