Assignment 9 for MTH G131: Fall 2006

Due date: Wednesday November 15.

Reading: Logan, Chapter 4; review integrals in a calculus text.

1). [Improper integral]

Suppose f is a real function on $(0,1]$, and suppose that f is Riemann-integrable on $[a,1]$ for every $a>0$. The improper integral is defined as

$$\int_0^1 f(x)dx = \lim_{a\to 0}\int_a^1 f(x)dx$$

if this limit exists and is finite. Construct a function f so that this limit exists and is finite, but does not exist with $|f|$ in place of f.

2). [More integrals]

a) Show that for any numbers x and y,

$$xy \le \frac{1}{2}\left(x^2+y^2\right)$$

b) Suppose that f and g are Riemann-integrable functions on $[a,b]$, and that

$$\int_a^b f(x)^2\,dx = \int_a^b g(x)^2\,dx = 1$$

Use the result of *(a)* to show that

$$\Big|\int_a^b f(x)g(x)dx\Big| \le 1$$

c) Suppose that F and G are Riemann-integrable functions on $[a,b]$. Use the result of *(b)* to derive the Cauchy-Schwarz inequality:

$$\Big|\int_a^b F(x)G(x)dx\Big| \le \Big(\int_a^b F(x)^2dx\Big)^{1/2}\Big(\int_a^b G(x)^2dx\Big)^{1/2}$$

3). [More more integrals]

The following integrals may or may not exist, due to singularities of the functions being integrated, and/or because the domain of integration is infinite. In each case, decide if the integral exists (you do NOT need to evaluate the integrals!). If you think the integral exists, you should find a positive integrable function that upper bounds the given function. If you think it does'nt exist, you should find a positive non-integrable function that lower bounds the given function. [Hint: look at how the function behaves when x is near the singularity, or when x is very large.]

$$\int_0^1 \frac{\sqrt{\sin x}}{x} dx, \quad \int_1^\infty (x + e^{-x})^{-1}\, dx$$

$$\int_0^1 (1 - x^2)^{-1/2}\, dx, \quad \int_1^\infty x^3\, (1 + e^x)^{-1}\, dx$$

4). [Laplace transforms]

Logan, p.138: #1, #2, #4