

**Problem Session for MTH G131: 10/12/06**

1). Find eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}$$

Use your result to find the solution of the following ODE:

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

2). For a  $2 \times 2$  matrix  $A$  with eigenvalues  $\lambda_1$  and  $\lambda_2$  show that

$$\text{Trace}(A) = \lambda_1 + \lambda_2, \quad \text{Det}(A) = \lambda_1 \lambda_2$$

3). Suppose the  $2 \times 2$  matrix  $A$  has distinct real eigenvalues  $0 < \lambda_1 < \lambda_2$ , with eigenvectors  $v_1$  and  $v_2$ . Show that for most vectors  $w$

$$\lim_{n \rightarrow \infty} (\lambda_2^{-1} A)^n w \rightarrow c v_2$$

for some constant  $c$ . For which vectors  $w$  is this result false?

4). Consider the  $2 \times 2$  matrix

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Compute  $A^2$ . Without doing any further calculations, what can you conclude about the eigenvalues of  $A$ ?

5). Consider the  $3 \times 3$  matrix

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Compute  $B^3$ . Without doing any further calculations, what can you conclude about the eigenvalues of  $B$ ?

6). Suppose the matrix  $A$  satisfies the equation

$$A^4 - 3A^3 + 6A^2 - I = 0$$

What can you conclude about the eigenvalues of  $A$ ?