Problem Session for MTH G131: 11/16/06

1). Use the Laplace transform to solve the initial value problems:

$$y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$$y'' - 2y' + 2y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1$$

2). Define

$$f(t) = \begin{cases} (\sin t)/t, & \text{for } t \neq 0 \\ 1 & \text{for } t = 0 \end{cases}$$

Find the Taylor series for f about $t = 0$. Assuming that the Laplace transform of this function can be computed term by term, show that

$$\mathcal{L}[f] = \arctan\left(\frac{1}{s}\right)$$

3). Suppose that $\mathcal{L}[f] = F(s)$, show that for constants a, b with $a > 0$,

$$\mathcal{L}^{-1}[F(as+b)] = \frac{1}{a}\, e^{-bt/a}\, f\big(\frac{t}{a}\big)$$

4). Find the inverse Laplace transform of

$$\frac{(s-2)\, e^{-s}}{s^2 - 4s + 3}$$

5). Find the solution of the IVP

$$y'' + 3y' + 2y = u_2(t), \quad y(0) = 1, \quad y'(0) = 0$$

6). Use the convolution integral to find the Laplace transform of

$$f(t) = \int_0^t (t-u)\, e^u\, du$$