MTH G131. Fall 2006

SOLUTIONS: Practice Problems 1

1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$\int y\,dy = \int \frac{x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + C.$$

2

$$x' = \frac{2t}{1+2x}, \quad x(2) = 0$$

$$\int (1+2x)\,dx = \int 2t\,dt$$

$$x + x^2 = t^2 + C \quad ; \; 0 = 4 + C$$

$$\boxed{x + x^2 = t^2 - 4}$$

$$x^2 + x + 4 - t^2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(4-t^2)}}{2} = \frac{-1 \pm \sqrt{4t^2 - 15}}{2}.$$

$$x(2) = 0 \Rightarrow \boxed{x(t) = -\frac{1}{2} + \sqrt{t^2 - \frac{15}{4}}}$$

Interval of existence $\frac{\sqrt{15}}{2} \le t < \infty$.

3.

$$\frac{du}{dt} = k(u - u_0)$$

a) $u(0) = u_0 \Rightarrow \boxed{u(t) = u_0}$ all t.

b)

$$\int \frac{du}{u - u_0} = \int k\,dt$$

$$\Rightarrow \ln(u - u_0) = kt + C$$

$$\boxed{u - u_0 = C e^{kt}}$$ valid for $u(0) > u_0$