

SUPERCRITICAL PITCHFORK BIFURCATION.

11  $x' = h - ax + \frac{x^2}{1+x^2} \quad (a > 0)$

a)  $h = 0$ : solve  $-ax + \frac{x^2}{1+x^2} = 0$ .

$x = 0$  or  $-a + \frac{x}{1+x^2} = 0 \Rightarrow x = a + ax^2$

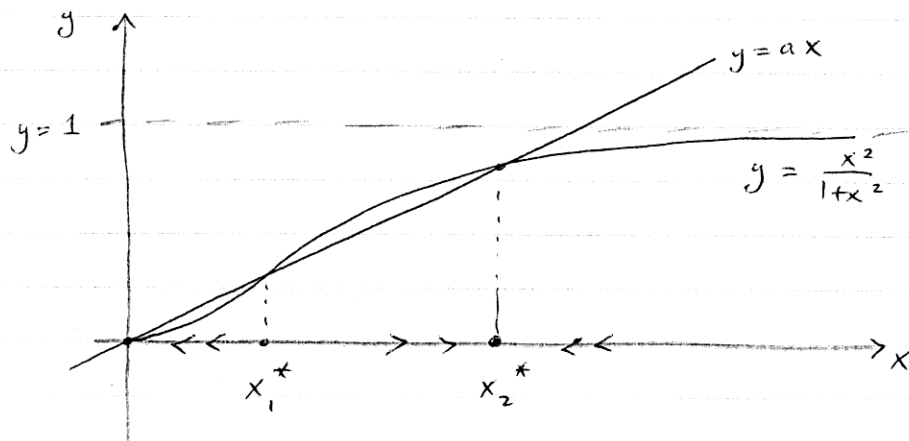
$ax^2 - x + a = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4a^2}}{2a}$

$\rightarrow \begin{cases} x_1^* = \frac{1}{2a} - \frac{\sqrt{1-4a^2}}{2a} \\ x_2^* = \frac{1}{2a} + \frac{\sqrt{1-4a^2}}{2a} \end{cases}$

$\Rightarrow$  two solutions if  $a < a_c = \frac{1}{2}$ .

b) Look at critical points for general  $h \geq 0$ .



$h = 0$