

$$\underline{S_8}: \lambda_1 = 2, \quad v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\lambda_2 = -1, \quad v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\boxed{5} \quad \frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}; \quad A - \lambda I = \begin{pmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{pmatrix}$$

$$\Rightarrow (2-\lambda)(-2-\lambda) + 3 = 0$$

$$\Rightarrow \lambda^2 - 4 + 3 = 0$$

$$\boxed{\lambda = \pm 1}$$

$$\boxed{\lambda = 1} \quad \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x - y = x \Rightarrow x = y$$
$$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\boxed{\lambda = -1} \quad \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = - \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x - y = -x \Rightarrow y = 3x$$
$$v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\Rightarrow$  general solution is

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = c_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$