

6

$$x' = 4x - 3y$$

$$y' = 6x - 7y$$

$$x(0) = 1, y(0) = -1$$

Linear : $A = \begin{pmatrix} 4 & -3 \\ 6 & -7 \end{pmatrix}$

E-values: $(4-\lambda)(-7-\lambda) + 18 = 0$

$$-28 - 4\lambda + 7\lambda + \lambda^2 + 18 = 0$$

$$\lambda^2 + 3\lambda - 10 = 0$$

$$(\lambda + 5)(\lambda - 2) = 0$$

$$\Rightarrow \lambda \in \{-5, 2\}$$

$\lambda = -5$

$$4x - 3y = -5x \Rightarrow 9x = 3y \quad ; y = 3x$$

$$\Rightarrow v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\lambda = 2$

$$4x - 3y = 2x \Rightarrow 2x = 3y \quad ; x = \frac{3}{2}y$$

$$\Rightarrow v_2 = 2 \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

General solution: $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^{-5t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

i.c.: $\begin{pmatrix} 1 \\ -1 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow$

$$1 = c_1 + 3c_2$$

$$-1 = 3c_1 + 2c_2$$

$$\underline{8 = 3c_1 + 9c_2}$$

$$4 = 0 + 7c_2$$

$$\boxed{c_2 = \frac{4}{7} \quad ; \quad c_1 = -\frac{5}{7}}$$