

⇒ solution is

$$x(t) = -\frac{5}{7} e^{-5t} + \frac{12}{7} e^{2t}$$

$$y(t) = -\frac{15}{7} e^{-5t} + \frac{8}{7} e^{2t}$$

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a)  $x' = 2x - y$

$$y' = 3x - 2y$$

$$A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$$

~~eigenvalues~~  $\text{tr } A = 0, \quad \det A = -4 + 3 = -1$

⇒ two real eigenvalues, opposite signs

⇒ UNSTABLE

b)  $x' = x - 5y$

$$y' = x - 3y$$

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}$$

$$\text{tr } A = -2, \quad \det A = -3 + 5 = 2$$

⇒ STABLE

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a)  $x' = x + x^2 + y^2 = x + f_1(x, y)$

$$y' = y - xy = y + g_1(x, y)$$

$f_1$  &  $g_1$  are quadratic ⇒ almost linear system

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \text{eigenvalues } \{+1\}$$

⇒ linear system is unstable

⇒ nonlinear system is unstable