b) $x' = (1+x)\sin y = (1+x)\left(y - \frac{y^3}{3!} + \cdots\right) = y + f_1(x,y)$

$y' = 1 - x - \cos y = 1 - x - \left(1 - \frac{y^2}{2!} + \cdots\right) = -x + g_1(x,y)$

f_1, g_1 are quadratic & higher order

$\Rightarrow$ almost linear system.

$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ eigenvalues $\{\pm i\}$

$\Rightarrow$ linear system stable, not asymptotically stable

$\Rightarrow$ no infos. about nonlinear system.

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$$m\frac{d^2u}{dt^2} + b\frac{du}{dt} + ku = 0.$$

$$\Rightarrow u'' + \beta u' + \alpha u = 0$$

$x = u$, $y = u'$

$$\Rightarrow \boxed{\begin{aligned} x' &= y \\ y' &= -\alpha x - \beta y \end{aligned}} \Rightarrow (0,0) \text{ is a critical point.}$$

Linear system $A = \begin{pmatrix} 0 & 1 \\ -\alpha & -\beta \end{pmatrix}$

e-values: $(-\lambda)(-\beta-\lambda) + \alpha = 0$

$$\lambda^2 + \lambda\beta + \alpha = 0$$

$$\lambda = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha}}{2}.$$