Assignment 6 for MTH G341: Spring 2005

Due date: Wednesday March 30.

Reading: Chapter 8.

Problems:

1). p.534: #4, #5

2). p.534: #10

3). A network server can process requests at the rate of 10 per second. It has a buffer that can store a maximum of 20 requests. Users can tolerate delays in response of up to 2 seconds. Making reasonable assumptions, calculate the maximum rate of requests that the server can handle.

4). Consider the queue $M/D/1$ where the arrivals are Poisson with rate λ, and the service time is constant and equal to d. Assume that $\rho = \lambda d < 1$. Show that the mean queue length at moments of departure in the steady state is $\rho(2-\rho)/2(1-\rho)$.

5). Derive the stationary distribution of the M/M/3 system without queue, i.e. Erlang's loss system, with arrival rate $\lambda = 2$, and service rate $\mu = 2$. Use your results to answer the following.
a) What proportion of arriving customers is lost?
b) Find the expected number of customers in the system, and the expected time spent in the system by an arriving customer.

6). Consider a network of three servers, all with infinite buffers. Requests arrive at servers 1,2,3 in accordance with Poisson processes with rates 5,10,15 respectively. The service times at the three servers are exponential with rates 10,50,100 respectively. A request which completes service at server 1 is equally likely to (a) go to server 2, (b) go to server 3, or (c) leave the network. A completed request at server 2 always goes to server 3. A completed request at server 3 is equally likely to either go to server 2 or to leave the network.

a) What is the average number of requests in the network? (assume that the network is in its stationary distribution).

b) What is the average time a request spends in the network?

c) What is the probability that a request which arrives at server 1 is processed three or more times by server 3?

d) Let T_1, T_2, T_3 be the time spent in the network by a request which arrives at server 1,2,3 respectively. Rank in increasing order the three expected values E[T_1], E[T_2], E[T_3].