MTH U481: Spring 2008: Prof. C. King

Assignment 5

Due date: Thursday February 14.

Reading: Sections 2.5, 3.3.

Problems:

1). p. 172: #4, #9, #11.

2). The pdf for a continuous random variable T is given as follows:

$$f_T(t) = \begin{cases} t & \text{if } 0 \le t \le 1, \\ 1 & \text{if } 1 \le t \le 1.5, \\ 0 & \text{otherwise.} \end{cases}$$

a) Sketch the graph of $f_T(t)$.
b) Calculate $P(0.5 \le T \le 1.25)$.

3). The pdf for a continuous random variable is

$$f_Y(y) = \begin{cases} c\,(2y - y^2) & \text{if } 0 \le y \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

where c is a constant.
a). Find the value of c.
b). Calculate $P(0 \le Y \le 1)$.
c). Find the cdf $F_Y(y)$.

4). A damaged lighthouse is located a distance L from a long wall. The lighthouse shines its beam toward the wall but in a random direction; more exactly if θ is the angle between the beam and the shortest line to the wall, then θ has a uniform distribution on the interval $[-\pi/2, \pi/2]$. Assume the length of the wall is infinite, and let X be the position where the beam strikes the wall, so that $Ran(X) = (-\infty, \infty)$ with $X = 0$ the point on the wall closest to the lighthouse.

a). Where the beam hits the wall, find the angle θ as a function of the position x.
b). Use the result of part (a) to find the values a, b, c so that

$$F_X(x) = P(X \leq x) = a + b \tan^{-1}(cx)$$

c). Use the result from part (b) to find the pdf of X.

5). p.185, #8.