MTH U481 : SPRING 2008: PRACTICE PROBLEMS FOR QUIZ 2

1). A biased coin has probability $p = 0.6$ of coming up Heads. The coin is tossed 5 times. Assume that the tosses are independent.

a). What is the probability that the first two tosses are Heads, and the last three are Tails?
Answer: $p^2(1-p)^3$

b). What is the probability that the first two tosses are Heads?
Answer: p^2

c). What is the probability that exactly two of the tosses come up Heads, in any order?
Answer: $10p^2(1-p)^3$

2). Based on past experience, the time taken by a randomly chosen student to complete a test in the course "Probability 1" varies between 35 minutes and 65 minutes. Let T denote the time taken for the test, and suppose that the pdf for T is

$$f(t) = \begin{cases} c(65-t) & \text{if } 35 \le t \le 65 \\ 0 & \text{else} \end{cases}$$

a) Find the value of the constant c.
Answer: $1/450$

b) Find the probability that the time taken is greater than 50 minutes.
Answer: $1/4$

c) Find the cdf of T (your answer should present different expressions for the cdf in three different intervals).
Answer: $F_T(t) = 1/450(65t - t^2/2)$ for $35 \le t \le 65$.

3). The cdf of a random variable X is given by

$$F_X(x) = \begin{cases} 1/3 + (2/3)(x+1)^2 & -1 \le x \le 0 \\ 0 & x < -1 \end{cases}$$

a) Calculate $F_X(1)$.
Answer: 1

b) Find $P(X > 1/3)$.
Answer: $1 - F_X(1/3) = 0$

c) Find $P(|X - 1/3| < 1)$.
Answer: $1 - F_X(-2/3) = 16/27$

4). A dart is thrown onto a square b units wide. Assume that the dart is equally likely to fall anywhere in the square. The random variable Z is the sum of the two coordinates of the point where the dart lands. (Take the origin of coordinates at the bottom left corner of the square, and the axes parallel to the sides of the square).

a) Find the range of Z.
Answer: $[0, 2b]$

b) Find the region in the square corresponding to the event $\{Z \leq z\}$, for all z in the range of Z.

c) Find $P(Z \leq z)$.
Answer: $z^2/2b^2$ for $0 \leq z \leq b$, $1 - (2b - z)^2/2b^2$ for $b \leq z \leq 2b$.

d) Find the pdf of Z.
Answer: z/b^2 for $0 \leq z \leq b$, $(2b - z)/b^2$ for $b \leq z \leq 2b$.

5). A continuous random variable X has the following pdf:

$$f_X(x) = \begin{cases} k(1 + x^2) & \text{for } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.
Answer: $k = 3/8$
(b) Compute $\mathrm{E}[X]$.
Answer: 0
(c) Compute $\mathrm{E}[X^2]$.
Answer: $2/5$
(d) Compute $\mathrm{VAR}[X]$.
Answer: $2/5$

6). pdf for X is $f(x) = c$ for $-1 \leq x \leq 1$, and $f(x) = c^2$ for $1 < x \leq 2$. Find c, calculate $E[X]$ and $E[X^2]$. [Answer: $c = \sqrt{2} - 1$, $3c^2/2$, $(2c + 7c^2)/3$]

7). pdf for X is $f(x) = k(1 - x^2)$ for $-1 \leq x \leq 1$. Find k, find $P(|X - 1/4| \geq 1/2)$. [Answer: $k = 3/4$, $23/64$]

8). cdf for X is $F(x) = x/4$ for $0 \leq x \leq 2$, and $F(x) = (x - 1)/2$ for $2 < x \leq 3$. Find $P(1 \leq X \leq 5/2)$, find the pdf, calculate $E[X]$. [Answer: $1/2$, $E[X] = 7/4$]

9). Roll two fair dice, and let X be the absolute value of the difference between the numbers on the dice. Find the pdf of X, and find the mean of X. [Answer: $E[X] = 70/36$]