

MTH U481 : SPRING 2009: MORE PRACTICE PROBLEMS FOR MIDTERM

1). Here is the joint pdf of two discrete r.v.'s X, Y :

$$P(X = 0, Y = -1) = 1/12, \quad P(X = 1, Y = -1) = 1/4, \quad P(X = 2, Y = -1) = 1/6$$

$$P(X = 0, Y = 1) = 1/4, \quad P(X = 1, Y = 1) = 1/6, \quad P(X = 2, Y = 1) = 1/12$$

Find the marginals, find expected values and standard deviations of X and Y , find covariance and correlation coefficient of X, Y .

Answer:

$$P(X = 0) = 1/3, \quad P(X = 1) = 5/12, \quad P(X = 2) = 1/4$$

$$P(Y = -1) = P(Y = 1) = 1/2$$

$$E[X] = 11/12, \quad E[Y] = 0, \quad VAR[X] = 0.576, \quad VAR[Y] = 1, \quad COV[X, Y] = -1/4, \quad \rho(X, Y) = -0.33.$$

2). A point (X, Y) is picked randomly and uniformly from the region with corners at $(0, 0), (1, 2), (3, 0), (3, 3)$. Find the joint pdf of X, Y , find the marginals of X and Y , and compute expected values of X and Y .

Answer: $f(x, y) = 1/6$ in the region, zero outside.

$$f_X(x) = \begin{cases} x/3 & 0 \leq x \leq 1 \\ (x+3)/12 & 1 \leq x \leq 3 \end{cases}$$

$$f_Y(y) = \begin{cases} (6-y)/12 & 0 \leq y \leq 2 \\ (6-2y)/6 & 2 \leq y \leq 3 \end{cases}$$

$$E[X] = 11/6, \quad E[Y] = 7/6$$

3). An airplane carries 150 bags. The bags have weights that are random, with mean 20 kg and standard deviation σ kg. Find the maximum value of σ so that the total weight of bags has a standard deviation not exceeding 50 kg.

Answer: 4.08 kg

4). There are N people at a Red Sox party, all wearing a Yankees hat. They throw the hats into the center of the room and then each person randomly selects a hat. Find the expected number of people who select their own hat.

Answer: 1

5). A lighthouse is distance L from a long wall (treat the wall as having infinite length). The lighthouse beam shines toward the wall and hits it at a point X (X is measured from the point closest to the lighthouse, so X ranges from $-\infty$ to $+\infty$). The angle of the beam θ is measured from the line perpendicular to the wall, so θ ranges from $-\pi/2$ to $+\pi/2$. Assuming that θ is random and uniform on $[-\pi/2, \pi/2]$, find the pdf of X .

Answer:

$$f_X(x) = \frac{L}{\pi} \frac{1}{x^2 + L^2}$$