

MTH U481 : SPRING 2009: PRACTICE PROBLEMS FOR QUIZ 2

1). A biased coin has probability $p = 0.6$ of coming up Heads. The coin is tossed 5 times. Assume that the tosses are independent.

a). What is the probability that the first two tosses are Heads, and the last three are Tails?

Answer: $p^2(1 - p)^3$

b). What is the probability that the first two tosses are Heads?

Answer: p^2

c). What is the probability that exactly two of the tosses come up Heads, in any order?

Answer: $10p^2(1 - p)^3$

2). Based on past experience, the time taken by a randomly chosen student to complete a test in the course "Probability 1" varies between 35 minutes and 65 minutes. Let T denote the time taken for the test, and suppose that the pdf for T is

$$f(t) = \begin{cases} c(65 - t) & \text{if } 35 \leq t \leq 65 \\ 0 & \text{else} \end{cases}$$

a) Find the value of the constant c .

Answer: $1/450$

b) Find the probability that the time taken is greater than 50 minutes.

Answer: $1/4$

c) Find the cdf of T (your answer should present different expressions for the cdf in three different intervals).

Answer: $F_T(t) = 1/450(65t - t^2/2) - (35)(95)/900$ for $35 \leq t \leq 65$.

3). The cdf of a random variable X is given by

$$F_X(x) = \begin{cases} 1/3 + (2/3)(x + 1)^2 & -1 \leq x \leq 0 \\ 0 & x < -1 \end{cases}$$

a) Calculate $F_X(1)$.

Answer: 1

b) Find $P(X > 1/3)$.

Answer: $1 - F_X(1/3) = 0$

c) Find $P(|X - 1/3| < 1)$.

Answer: $1 - F_X(-2/3) = 16/27$

4). A continuous random variable X has the following pdf:

$$f_X(x) = \begin{cases} k(1+x^2) & \text{for } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k .

Answer: $k = 3/8$

(b) Compute $E[X]$.

Answer: 0

(c) Compute $E[X^2]$.

Answer: $2/5$

5). The cdf of Y is $F_Y(y) = 1 - y^{-4}$ for $y \geq 1$. Plot the cdf, find $P(Y \geq 2)$, find $P(k < Y \leq k+1)$ for any $k \geq 1$, and find the pdf.

Answer: $1/16$, $k^{-4} - (k+1)^{-4}$, $4y^{-5}$ for $y \geq 1$

6). The pdf of X is $f(x) = c$ for $-1 \leq x < 0$ and $f(x) = 2c$ for $0 \leq x \leq 1$. Find c , find $E[X]$ and $E[|X|]$.

Answer: $c = 1/3$, $1/6$, $1/2$

7). X is uniform on $[0, 1]$ and $Y = e^X$. Find the mean of Y .

Answer: $E[Y] = e - 1$

8). Roll two fair dice, and let X be the absolute value of the difference between the numbers on the dice. Find the pdf of X , and find the mean of X .

Answer: $E[X] = 70/36$

9). cdf for X is $F(x) = x/4$ for $0 \leq x \leq 2$, and $F(x) = (x-1)/2$ for $2 < x \leq 3$. Find $P(1 \leq X \leq 5/2)$, find the pdf, calculate $E[X]$.

Answer: $1/2$, $E[X] = 7/4$