

**MTH U481 : SPRING 2009: PRACTICE PROBLEMS FOR QUIZ 3**

1). A recent poll showed that 45% of Americans believe the human race was created within the last 10,000 years. You plan to hold a party to celebrate Charles Darwin's birthday, and you want to invite enough people so that with probability at least 95% the number of people believing the statement above will be in the minority. Assuming guests are randomly sampled from the population, how many should you invite?

*Answer:* at least 268.

2). In a poll of 1000 voters,  $X$  said yes to a clean elections law. In an independent poll of 2000 voters,  $Y$  said yes to the same question. Let  $p$  be the proportion of the whole population which would say yes to this question.

a). Let  $\hat{p} = (2X + Y)/4000$ . Find the mean square error MSE of  $\hat{p}$ .

*Answer:*  $\text{MSE} = \frac{3p(1-p)}{8000}$ .

b). Can you propose a different estimator with a smaller MSE?

*Answer:*  $\frac{X+Y}{3000}$ .

3). The random variables  $X_1, X_2, \dots, X_n$  are independent and all have the same pdf, namely

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the MLE for  $\lambda$ . [Hint: write down the likelihood function  $L(\lambda)$ , take its logarithm, and compute the derivative with respect to  $\lambda$ ].

*Answer:*  $\frac{2n}{\sum_{i=1}^n x_i}$ .

4). The gasoline consumption of a typical SUV is modeled by a normal random variable whose mean is unknown, but whose standard deviation is assumed to be 8 miles per gallon. A random sampling of seven SUV's produced an average consumption rate of 15 miles per gallon (and several angry letters from the manufacturers). Construct a 95% two-sided confidence interval for the mean consumption of an SUV.

*Answer:* [9.1, 20.9].

5). In a recent poll 1000 people were asked the question "Is there life on Mars?". A surprisingly large number (754) answered "yes" (including one who claimed first hand knowledge). Construct a 99% confidence interval for the fraction of the population that believes in Martians.

*Answer:* [0.719, 0.789].

6). The mean length of pregnancy for a house cat is 60 days. A sample of 10 feral cats finds a mean length of 53 days with a standard deviation of 7. Test at the 5% level of confidence to see if the mean length for feral cats is less.

a) Give the null and alternate hypotheses.

*Answer:* let  $\mu$  be mean length of pregnancy for feral cats:

$$H_0 : \mu = 60, \quad H_1 : \mu < 60$$

b) Find the test statistic, decide if you accept or reject the null hypothesis, and explain what that means here.

*Answer:*  $TS = -3.16$ ,  $t_{0.05,9} = 1.8331$ ; reject  $H_0$ ; evidence supports conclusion that feral cats have shorter mean pregnancy time.

7). A manufacturer claims that its widgets have a mean lifetime of 110 hours. In fact the widget lifetime has mean  $\mu = 105$  hours and standard deviation  $\sigma = 15$  hours. Suppose 100 widgets are tested and their lifetimes are measured. Find the probability that the resulting 95% confidence interval for  $\mu$  will contain the value 110.

*Answer:* 0.0853