

3) p. 299)

481 sp 08, Assjamat #9

SOLUTIONS

#1 a) $\Phi(1.33) - \Phi(-0.44)$

b) $\Phi(0.94)$

#2 b) $\Phi(-0.11) - \Phi(-0.64)$

c) $1 - \Phi(-1.06)$

#8. $X = \#$ usable units.

$$X \sim \text{Bin}(n, p) \quad n = 1600, \quad p = 0.8$$

Want $P(1260 \leq X \leq 1310)$.

Normal approx: $X = \sigma Z + \mu$.

$$\mu = E[X] = np = 1280$$

$$\sigma = \text{STD}[X] = \sqrt{np(1-p)} = 16$$

$$\Rightarrow P\left(\frac{1260-1280}{16} \leq Z \leq \frac{1310-1280}{16}\right) = \Phi(1.875) - \Phi(-1.25)$$

#14. $X = \#$ hot days brought

$$X \sim \text{Bin}(n, p) \quad n = 42,200 \quad p = 0.38$$

$N = \#$ hot days ordered

Want $P(X > N) \leq 0.2$.

Normal approx: $X = \sigma Z + \mu$

$$\mu = np = 16036$$

$$\sigma = \sqrt{np(1-p)} = 99.71$$

$$\Rightarrow P(X > N) = P\left(Z > \frac{N - 16036}{99.71}\right) = 0.2$$

$$\Rightarrow \Phi\left(\frac{N - 16036}{99.71}\right) = 1 - 0.2 = 0.8$$

$$\Rightarrow \frac{N - 16036}{99.71} = 0.84 \Rightarrow N \geq 16120.$$

4 (p. 314)

#21 $X = \# \text{ miles from tree}$

$$X \sim N(\mu, \sigma^2) \quad \mu = 30,000 \quad X = \sigma Z + \mu$$
$$\sigma = 5000$$

$$P(X \geq 25,000) = P(Z \geq -1) = 1 - \Phi(-1) = 1 - 0.1587$$
$$= 0.8413$$

\Rightarrow (NO)

#23 $X = \text{tomorrow's demand}$

$$X \sim N(\mu, \sigma^2) \quad \mu = 20,000 \quad \sigma = 5000; \quad X = \sigma Z + \mu$$

$$P(X > 30,000) = P(Z > 2) = 1 - \Phi(2)$$

#26 $X = \text{area of tube} \sim N(\mu, \sigma^2)$

$$\mu = 12.5, \quad \sigma = 0.2; \quad X = \sigma Z + \mu$$

Right size: $P(12 \leq X \leq 13) = P(-2.5 \leq Z \leq 2.5) = \Phi(2.5) - \Phi(-2.5)$

$$\Rightarrow \# \text{ wrong-sized tubes} = 1000 (1 - \Phi(2.5) + \Phi(-2.5))$$

#30 $X = \text{weight} \sim N(\mu, \sigma^2)$

$$\mu = \frac{103.5 + 144.5}{2} = 124$$

$$X = \sigma Z + \mu$$

$$P(103.5 \leq X \leq 144.5) = 0.8$$

$$\Rightarrow P\left(-\frac{20.5}{\sigma} \leq Z \leq \frac{20.5}{\sigma}\right) = 0.8$$

$$\Rightarrow \Phi\left(-\frac{20.5}{\sigma}\right) = 0.1 \Rightarrow \frac{20.5}{\sigma} = 1.28$$

$$\Rightarrow \sigma = 16.02$$