MTH 481 Sp 08

Assignment #10

$\boxed{\text{SOLUTIONS}}$

$\boxed{2}$. (p 279)

#1. $X = \#$ errors in report

$X \sim Bin(n, p)$ $\quad n = 6000, \quad p = \frac{1}{3250}$.

Exact: $P(X=0) = (1-p)^{6000} = 0.1578$

Poisson: $\lambda = np = \frac{6000}{3250}$

$$P(X=0) = e^{-\lambda} = 0.1578$$

#7 $X = \#$ bags lost

$X \sim Bin(n, p)$ $\quad n = 120, \quad p = \frac{1}{200}$

$$\lambda = np = 0.6$$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - e^{-\lambda} - \lambda e^{-\lambda} = 0.122$$

#34 $\bar{Y} = \frac{1}{n} \sum Y_i \Rightarrow E[\bar{Y}] = E[Y_i] = 2$

$$STD[\bar{Y}] = \frac{1}{\sqrt{n}} STD[Y_i] = \frac{2}{\sqrt{n}}$$

$$\bar{Y} = \frac{2}{\sqrt{n}} Z + 2$$

$$\Rightarrow P(1.9 \leq \bar{Y} \leq 2.1) = P\left(-\frac{0.1\sqrt{n}}{2} \leq Z \leq \frac{0.1\sqrt{n}}{2}\right) = 0.99$$

$$\Rightarrow \Phi\left(-\frac{0.1\sqrt{n}}{2}\right) = 0.005$$

$$\Rightarrow \frac{0.1}{2}\sqrt{n} = 2.57 \Rightarrow n \geq 2642$$

3\. $X_i \sim$ Poisson $\lambda = 1 \Rightarrow E[X] = STD[X] = 1.$

$Y = X_1 + X_2 + \dots + X_{10} \Rightarrow E[Y] = 10, \quad STD[Y] = \sqrt{10}$

Normal approx: $Y = \sigma Z + \mu = \sqrt{10} Z + 10.$

$P(Y \geq 15) = P\left(Z \geq \frac{5}{\sqrt{10}}\right) = 1 - \Phi\left(\frac{5}{\sqrt{10}}\right).$

4\. $VAR[XY] = E[(XY)^2] - (E[XY])^2$

$= E[X^2]\,E[Y^2] - (E[X]\,E[Y])^2$ ($\because$ independent)

$= (\sigma_x^2 + \mu_x^2)(\sigma_y^2 + \mu_y^2) - (\mu_x \mu_y)^2$

5\. X_i = weight of i^{th} potato $\sim N(0.2, (0.05)^2)$

a) W = weight of bag $\sim N(20(0.2), 20(0.05)^2)$

b) $\bar{X} = \frac{1}{20} \sum X_i \sim N\left(0.2, \frac{1}{20}(0.05)^2\right).$

c) $P(W > 4.3) = P\left(Z > \frac{4.3 - 4.0}{\sqrt{20(0.05)^2}}\right)$

$= 1 - \Phi(1.34).$