MTH 581: Sp 08

Assignment #1: SOLUTIONS

$\boxed{1}$ (p. 92 #4)

(a) # plates $= (26)^2 (10)^4$

(b) # plates $= (26)^2 (10)(9)(8)(7)$

(c) # plates $= (26)^2 \left[(10)^4 - 1\right]$.

$\boxed{2}$ (p. 105, #36)

(a) # ways $= \underbrace{\binom{8}{3}}_{\text{Red books}} \cdot \underbrace{\binom{5}{3}}_{\text{Blue books}} \cdot \underbrace{\binom{2}{2}}_{\text{Brown books}}$

(b) # ways $= 8!$

(c) select 3 slots for Red first, then order the others

$\Rightarrow$ # ways $= \binom{8}{3} \cdot 5!$

$\boxed{3}$ (p. 112)

#(53)

(a) $\binom{9}{4}$

(b) $\boxed{\text{5 Men}} \xrightarrow{\text{choose 2}}$ $\quad$ $\boxed{\text{4 Women}} \xrightarrow{\text{choose 2}}$

$\Rightarrow \binom{5}{2} \cdot \binom{4}{2}$

(c) $\boxed{5 \text{ Men}}$ $\xrightarrow{\text{choose } k}$ $\boxed{4 \text{ Women}}$ $\xrightarrow{\text{choose } 4-k}$

$$\Rightarrow \binom{5}{1}\binom{4}{3} + \binom{5}{2}\binom{4}{2} + \binom{5}{3}\binom{4}{1}$$

(#55) $\binom{10}{5}$ (when first team is selected, second is also fixed)

(#59) $\boxed{2n}$ $\xrightarrow{\text{select } n}$ same as $\boxed{n}$ $\xrightarrow{\text{select } k}$ $\boxed{n}$ $\xrightarrow{\text{select } n-k}$

$$\Rightarrow \binom{2n}{n} = \binom{n}{n}\binom{n}{0} + \binom{n}{n-1}\binom{n}{1} + \binom{n}{n-2}\binom{n}{2} + \cdots \binom{n}{0}\binom{n}{n}$$

$$= \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2$$

$\boxed{4}$ (p.122)

(#4). $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

P(A): $\boxed{4 \text{ Aces}}$ $\xrightarrow{\text{select } 4}$ $\boxed{48 \text{ cards}}$ $\xrightarrow{\text{select } 9}$ select 13 $\Rightarrow$ # ways $= \binom{48}{9}$

Since total # ways is $\binom{52}{13}$ we get

$$P(A) = \binom{48}{9} \Big/ \binom{52}{13}$$

Same for B: $P(B) = \binom{48}{9} \Big/ \binom{52}{13}$

For $A \cap B$, same reasoning gives

$$P(A \cap B) = \binom{44}{5} \Big/ \binom{52}{13}$$

Hence $P(A \cup B) = \dfrac{2\binom{48}{9} + \binom{44}{5}}{\binom{52}{13}}$.

(#11). Total number of possibilities is 7^7 (each person has 7 choices). If they each chose differently then number of possibilities is $7!$ Hence

$$P(\text{all on different floors}) = \frac{7!}{7^7}.$$

Clearly $P(\text{all on same floor}) = \dfrac{7}{7^7}$.

(#14) By reasoning similar to #11, get

$$P(\text{each face appears}) = \frac{6!}{6^6}$$