MTH U581: Spring 2008: Prof. C. King

Assignment 3

Due date: Monday, February 18.

Problems:

Let $S_n = X_1 + X_2 + \cdots + X_n$ be a random walk on the integers starting at zero, which at each step can move one step left (with probability q), stay where it is (with probability r) or move one step right (with probability p). So X_i takes values $-1, 0, +1$ with probabilities q, r, p respectively, and all the $\{X_i\}$ are independent.

1. Let

$$N_1 = \min\{n : S_{n-1} = S_n\}$$

Find the distribution of N_1, and compute $E[N_1]$ and $E[S_{N_1}]$. [Hint: remember Wald's theorem]

2. Let

$$N_2 = \min\{n : S_n = S_{n+1}\}$$

Use the results from **1)** to find the distribution of N_2, and compute $E[N_2]$ and $E[S_{N_2}]$. Does the statement of Wald's theorem hold? If not why not?

3. Consider now the symmetric random walk starting at zero, so $p = q = 1/2$ and $r = 0$. Let

$$N_3 = \min\{n : S_n = -1\}$$

Find $E[N_3]$ (use either method from class).

4. Same random walk as in **3)**, let

$$N_4 = \min\{n > N_3 : S_n = -1\}$$

So N_4 is the second time the walk hits the point -1. Find $E[N_4]$.

5. A fair coin is repeatedly tossed. Find the expected number of tosses until the first Head appears. Then use this answer to find the expected number of tosses until the first two consecutive Heads appear.