MTH U581: Spring 2008: Prof. C. King

Assignment 9

Due date: Thursday, April 10.

1) Consider a branching process with mean family-size μ. Letting X_n denote the population in the n^{th} generation, show that for $m \leq n$

$$E[X_m X_n] = \mu^{n-m} \, E[X_m^2]$$

2) Events occur according to a Poisson process with rate $\lambda = 2$ per hour.

(a) What is the probability that no events occur between 8pm and 9pm?

(b) Starting at noon, what is the expected time at which the fourth event occurs?

(c) What is the probability that two or more events occur between 6pm and 8pm?

3) Pulses arrive at a Geiger counter in accordance with a Poisson process at a rate of three arrivals per minute. Each particle arriving at the counter has a probability 2/3 of being recorded. Let $X(t)$ denote the number of pulses recorded by time t (in minutes). Find $P(X(t) = 0)$ and calculate $E[X(t)]$.

4) Suppose that people arrive at a bus stop in accordance with a Poisson process with rate λ. Let $N(t)$ denote the number of arrivals by time t. The bus departs at time t. Let X denote the total amount of waiting time of all those who get on the bus at time t (so X is the sum of the waiting times of all passengers who get on the bus).

(a) Find the conditional expectation $E[X|N(t)]$.

(b) Calculate $E[X]$.

5) Let T_n be the time of the n^{th} arrival in a Poisson process N with intensity λ, and define the excess lifetime process $E(t) = T_{N(t)+1} - t$, being the time one must wait subsequent to t before the next arrival. Show by conditioning on T_1 that

$$P(E(t) > x) = e^{-\lambda(t+x)} + \int_0^t P(E(t-u) > x) \lambda e^{-\lambda u} du$$

Solve this integral equation to find the distribution function of $E(t)$.