

MTH U581 : SPRING 2008: PRACTICE PROBLEMS FOR TEST 1

1). n keys are placed randomly into n empty boxes.

(i) Find the probability that (a) all boxes contain exactly one key, (b) exactly one box remains empty.

(ii) Use Stirling's formula to approximate your answers in (i) when n is large.

2). A closet contains n pairs of shoes. If $2r$ shoes are chosen at random (with $2r < n$), what is the probability that there will be (a) no complete pair, (b) exactly one complete pair, (c) exactly two complete pairs among them?

3). Let S_n be the position of the usual random walk after n steps, with probabilities p, q of right and left steps respectively. Find $P(S_9 = 5)$ and $P(S_{20} = -2)$.

4). Use l'Hopital's rule to evaluate

$$\lim_{q/p \rightarrow 1} \frac{(q/p)^{100} - 1}{(q/p)^{400} - 1}$$

[Hint: write $x = q/p$ and compute the limit as $x \rightarrow 1$]

5). Let $Y_n = a + X_1 + X_2 + \cdots + X_n$ where $\{X_i\}$ are independent, with the following pdf:

$$P(X_i = 1) = r, \quad P(X_i = 0) = t, \quad P(X_i = -1) = s$$

and $r + s + t = 1$. Find the probability that Y_n hits $a + b$ before it hits 0. [Hint: imitate the method used in class for the case with $t = 0$].