MTH 4581 Spring 2009

Assignment #1 ; SOLUTIONS

1 (p. 39)

(26)
$$1 = P(\text{higher}) + P(\text{lower}) + P(\text{same})$$
$$= 2P(\text{higher}) + P(\text{same}) = 2P(\text{higher}) + \left(\frac{3}{51}\right)$$
$$\Rightarrow P(\text{higher}) = \frac{24}{51}.$$

2 (p. 114)

(7)
$$b(n,p,j) = \binom{n}{j} p^j (1-p)^{n-j}$$
$$= \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j}$$
$$= \frac{n!}{(j-1)!(n-j+1)!} p^{j-1} (1-p)^{n-j+1} \left(\frac{n-j+1}{j}\right)\left(\frac{p}{1-p}\right)$$
$$= \frac{n-j+1}{j} \frac{p}{1-p} b(n,p,j-1).$$

$$\Rightarrow \frac{b(n,p,j)}{b(n,p,j-1)} = \frac{p}{1-p}\left(\frac{n-j+1}{j}\right)$$ ← RHS is decreasing in j.

$\Rightarrow b(n,p,j)$ reaches a max., then decreases.

Solve $$\frac{b(n,p,j)}{b(n,p,j-1)} = 1 \Leftrightarrow p(n-j+1) = (1-p)j \Leftrightarrow j = (n+1)p.$$

so maximum occurs at $j = [(n+1)p]$ (integer part)

13 want: $\max_k \binom{2n}{k} = \binom{2n}{n}$.

From #7: $p = \frac{1}{2}$, $n \to 2n$

$\max_k b(2n, \frac{1}{2}, k)$ at $k = [(2n+1)\frac{1}{2}] = [n + \frac{1}{2}] = n$. ✓

3 (p. 114)

22. like method described

$\Rightarrow$ put 6 letters and 2 partitions in any order into 8 slots.

$\Rightarrow$ # ways $= \binom{8}{2} = \frac{(8)(7)}{(2)} = 28.$

23 As above: have r objects and $(n-1)$ partitions

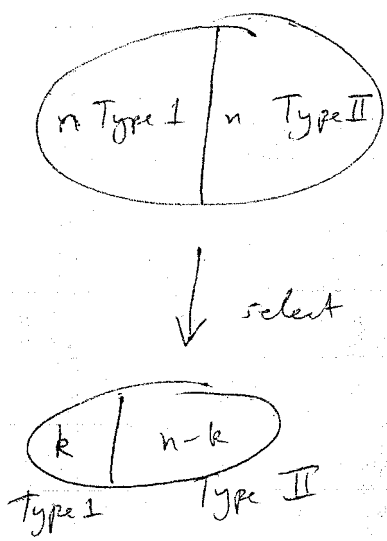
$\Rightarrow$ have $n+r-1$ slots.

select $n-1$ of these for the partitions

$\Rightarrow$ # ways $= \binom{n+r-1}{n-1} = \binom{n+r-1}{r}$.

4 (p. 114)

35



$$\# \text{ways} = \binom{n}{k}\binom{n}{n-k} = \binom{n}{k}^2.$$

Sum over k

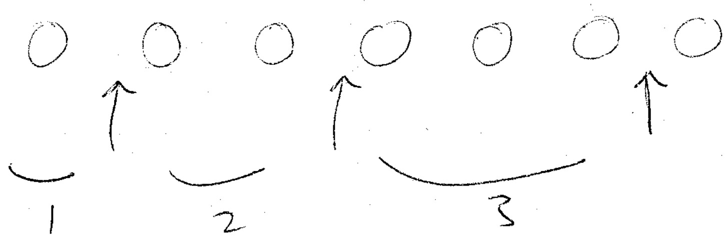
$\Rightarrow$ get all subsets of size n

$$\Rightarrow \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$

Put n slots in a row

(36). a) Have n available gaps to the right of each slot. Select j of these gaps, then j-tuple is succession of number of slots between gaps.

e.g.



$\Rightarrow \quad n = 7$

$j = 3$

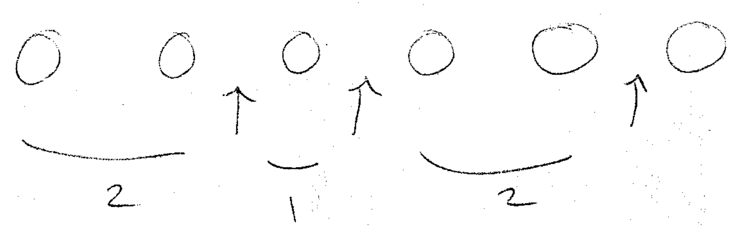
j-tuple : $(1, 2, 3)$

so $\# \text{ways} = \binom{n}{j}$.

b) Put $n-j$ slots in a row.

Repeat construction above, but now add 1 to every integer in j-tuple.

e.g



$j = 3$

$n-3 = 6 \Rightarrow n = 9$

j-tuple : $(3, 2, 3)$

$$\Rightarrow \# \text{ways} = \binom{n-j}{j}$$

$$\Rightarrow P(j\text{-tuple contains at least one } 1) = 1 - \frac{\binom{n-j}{j}}{\binom{n}{j}}$$