

**MTH U581: Spring 2008: Prof. C. King**

**Assignment 3**

**Due date:** Monday, February 2.

**Reading:** Chapter 6.

**Problems:**

1. A biased coin has probability  $p$  of coming up Heads. The coin is tossed repeatedly. Let  $N$  be the number of tosses until Heads appears for the first time. By conditioning on the outcome of the first toss, find the expected values of  $N$  and  $N^2$ . Use this to find the variance of  $N$ .

2. Let  $X_1, X_2, \dots$  be independent and identically distributed with mean  $E[X]$  and variance  $VAR[X]$ . Let  $N$  be a nonnegative integer-valued random variable independent of the  $X_i$ 's. Show that

$$VAR\left[\sum_{i=1}^N X_i\right] = E[N] VAR[X] + (E[X])^2 VAR[N]$$

3. A rat is trapped in a maze. Initially he must choose one of two directions. If he goes to the right, then he will wander around in the maze for three minutes and will then return to his initial position. If he goes to the left, then with probability  $1/3$  he will depart the maze after two minutes of traveling, and with probability  $2/3$  he will return to his initial position after five minutes of traveling. Assuming that the rat is at all times equally likely to go to the left or the right, what is the expected number of minutes that he will be trapped in the maze?

4. A die is repeatedly rolled until the same number appears  $k$  consecutive times. If  $N$  denotes the number of rolls, show that

$$E[N] = \frac{6^k - 1}{5}$$

5. A coin having probability  $p$  of coming up Heads is tossed until two of the most recent three tosses are Heads. Let  $N$  denote the number of tosses needed (if the first two tosses are Heads then  $N = 2$ ). Find  $E[N]$ .