

MTH U581 : SPRING 2009: PRACTICE PROBLEMS FOR TEST 1

- 1). The letters A,C,C,L,L,S,U,U are re-arranged at random. Find the probability that they form the word CALCULUS.
- 2). n keys are placed randomly into n empty boxes. Find the probability that (a) all boxes contain exactly one key, (b) exactly one box remains empty.
- 3). A closet contains n pairs of shoes. If $2r$ shoes are chosen at random (with $2r < n$), what is the probability that there will be (a) no complete pair, (b) exactly one complete pair, (c) exactly two complete pairs among them?
- 4). A fair coin is tossed n times. Find the conditional probability of a Head on any specified toss, given that there is a total of k Heads in the n tosses.
- 5). Prove or give a counterexample: if $P(B|A) > P(B)$ then $P(B^c|A) < P(B^c)$.
- 6). A coin, having probability p of landing Heads, is continually tossed until at least one Head and one Tail have been tossed. Find the expected number of tosses needed. [Hint: condition on the outcome of the first toss].
- 7). There are three coins in a barrel. These coins, when tossed, will come up Heads with respective probabilities 0.3, 0.5, 0.7. A coin is randomly selected from among these three and is then tossed ten times. Let N be the number of Heads obtained on the ten tosses. Find $P(N = n)$ for $n = 0, 1, 2, \dots, 10$. [Hint: condition on the coin selected].
- 8). Let X and Y be independent random variables with the same pdf, namely uniform on the set $\{0, 1, 2, 3\}$. Find the moment generating function of $2X + 3Y - 4$.