

**MTH 7241: Fall 2009: Prof. C. King**

**Assignment 2**

**Due date:** Monday, September 28.

**Reading:** additional notes on webpage, Chapter 1 titled ‘Elementary Probability’.

**Problems:**

- 1) A biased coin has probability  $p$  to come up Heads when tossed. The coin is repeatedly tossed. Let  $N_{HH}$  be the number of tosses until the first occurrence of two consecutive Heads. Find  $E[N_{HH}]$ . [Hint: condition on  $N_H$ , the number of tosses until the first occurrence of Heads].
- 2) Assuming that you worked out Problem (1) correctly, try to extend the technique and calculate  $m_k = E[N_{H\dots H}]$ , where  $N_{H\dots H}$  is the number of tosses until the first occurrence of  $k$  successive Heads.
- 3) A system has two components placed in series so that the system fails if either of the two components fails. The second component is twice as likely to fail as the first. If the two components operate independently, and if the probability that the entire system fails is 0.28, find the probability that the first component fails.
- 4) Sam writes to Fred and does not receive an answer. Assuming that one letter in  $n$  is lost in the mail, find the chance that Fred received the letter. You should assume that Fred would have answered the letter if he received it.
- 5) A life insurance application asks the question “Are you a smoker?” The percentage of smokers in the general population is 15%. Furthermore, 40% of applicants who are smokers say that they are non-smokers on the application, but none of the non-smokers lie on the application. What proportion of applicants who say they are non-smokers are actually non-smokers?
- 6) A manuscript is sent to a typing firm consisting of typists A,B and C. If it is typed by A then the number of errors is a Poisson random variable with mean 2.6; if it is typed by B then the number of errors is a Poisson random variable with mean 3; if it is typed by C then the number of errors is a Poisson random variable with mean 3.4. Let  $X$  denote the number of errors in the typed manuscript. Assume that each typist is equally likely to do the work. Find the mean and variance of  $X$ .