MTH 6724I

Assignment 2 : $\boxed{\text{SOLUTIONS}}$

1). $E[N_{HH} \mid N_H = k] = p(k+1) + (1-p)(k+1+E[N_{HH}])$

$$= k+1+(1-p)E[N_{HH}]$$

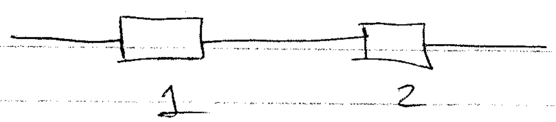
$$\Rightarrow E[N_{HH} \mid N_H] = N_H + 1 + (1-p)E[N_{HH}]$$

$$\Rightarrow E[N_{HH}] = E[N_H] + 1 + (1-p)E[N_{HH}]$$

$$\Rightarrow E[N_{HH}] = \frac{1}{p} + \frac{1}{p}E[N_H] = \frac{1}{p} + \frac{1}{p^2}$$

2). same method ; $m_k = \frac{1}{p} + \frac{1}{p^2} + \cdots + \frac{1}{p^k}$.

3)



$p_2 = 2p_1$

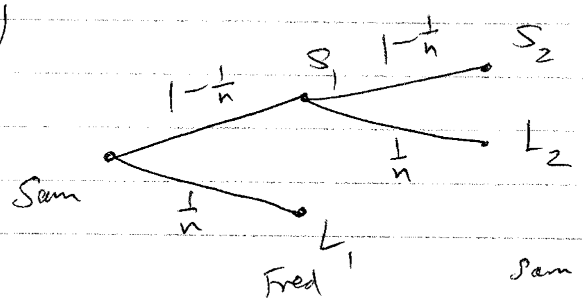
$$P(F_1 \cup F_2) = 0.28 = P(F_1) + P(F_2) - P(F_1)P(F_2)$$

$$= p_1 + p_2 - \cancel{2}\, p_1 p_2$$

$$0.28 = 3p_1 - \cancel{3/2}\, 2p_1^2$$

$$\Rightarrow p_1 = 0.1$$

4)



S = success

L = lost in mail

(2)

Want $P(S_1 \mid L_1 \cup L_2)$

$$= 1 - P(L_1 \mid L_1 \cup L_2)$$

$$= 1 - \frac{P(L_1)}{P(L_1) + P(L_2)} \qquad (L_1, L_2 \text{ disjoint})$$

$$= \frac{P(L_2)}{P(L_1) + P(L_2)} \qquad : P(L_2) = P(L_2 \mid S_1) P(S_1) = \frac{1}{n}\left(1 - \frac{1}{n}\right)$$

$$= \frac{\frac{1}{n}\left(1-\frac{1}{n}\right)}{\frac{1}{n} + \frac{1}{n}\left(1-\frac{1}{n}\right)}$$

$$= \frac{1-\frac{1}{n}}{2-\frac{1}{n}} = \frac{n-1}{2n-1}.$$

5) S: person is smoker.
F: person says they are non-smoker

given: $P(F \mid S) = 0.4$ $\qquad P(S) = 0.15$

$P(F \mid S^c) = 1$

Want: $$P(S^c \mid F) = \frac{P(F \mid S^c)\, P(S^c)}{P(F \mid S^c) P(S^c) + P(F \mid S) P(S)}$$

$$= \frac{1 \cdot (0.85)}{1 \cdot (0.85) + (0.4)(0.15)}$$

$$= \frac{85}{91}$$

6) $E[X] = E[X|A]\,P(A) + E[X|B]\,P(B) + E[X|C]\,P(C)$

$= (2.6)\,\frac{1}{3} + (3)\left(\frac{1}{3}\right) + (3.4)\left(\frac{1}{3}\right)$

$= 3$

Note: Poisson $\Rightarrow$ $E[X^2] = VAR[X] + E[X^2] = \lambda + \lambda^2$

$\Rightarrow E[X^2] = E[X^2|A]\,P(A) + E[X^2|B]\,P(B) + E[X^2|C]\,P(C)$

$= \left(2.6 + (2.6)^2\right)\frac{1}{3} + \left(3 + 3^2\right)\frac{1}{3} + \left(3.4 + (3.4)^2\right)\frac{1}{3}$

$= 12.11$

$\Rightarrow VAR[X] = E[X^2] - E[X]^2 = 3.11$