To verify this, we shall describe a general rule for constructing a route that passes along all edges of a graph just once in each direction. We begin our walk along some edge $e_0 = a_0a_1$ from an arbitrary vertex a_0. We mark this edge with a little arrow at a_0 to indicate which direction we have taken. We proceed successively to other vertices; the same vertex may be visited several times. At a_1, and each time later when a vertex is reached, we leave an arrow on the edge to indicate the direction of arrival. In addition, the first time we arrive at a new vertex we mark the entering edge specially so that it can be recognized later.

From each vertex we always exit along unused directions, either along edges which have not previously been traversed or along edges which have been marked as arrival edges; only when there are no other choices may we use the first entering edge as an exit.

We continue this winding walk as far as it is possible. At any vertex there are just as many possibilities for an exit as for an entry. As a consequence, the process can stop only at the initial vertex a_0. It remains to establish that at each vertex all edges have been traversed in both directions.

At a_0 this is simple, for all exit edges must have been used (since otherwise we could have gone further); hence all entering edges have been used, since there are just as many of these. In particular, the edge $e_0 = a_0a_1$ has been covered in both directions. But this means that all exits at a_1 have also been used, since the first entering edge should only be followed as a last resort. The same reasoning applies to the next edge $e_1 = a_1a_2$ and the next vertex a_2, and so on. In this manner we find that at all vertices we have reached all edges are covered in both directions. Since our graph is connected, this means that the whole graph has been traversed.

This method of passing through all edges of a graph may be used for many purposes. It may be used for finding a way out of a maze or a labyrinth, and should you by chance be lost in a cave you may give it a try.

Fig 13 A scheme for traversing all edges of a graph exactly twice, once in each direction.