MTH U341 Conservative vector fields homework
Due Monday, Nov. 17

Let $\vec{F}$ be the vector field in 3-space

$$\vec{F}(x,y,z) = <y^2+z^2, 2xy+z^2+3y^2, 2yz-2z+2xz>.$$

1. Find a potential function for $\vec{F}$.

2. Let C be the curve parametrized by

$$\vec{r}(t) = <t^5 - \sqrt{t(1-t^2)}, \sin(\pi t) + 2\cos(\pi t), e^{t^3}\sin(\pi t) + 2t>; \quad 0 \le t \le 1.$$

Find $\int_C \vec{F} \cdot d\vec{r}$.

3. Is $\vec{F}$ path-independent? In other words, if A and B are points in 3-space and C_1 and C_2 are two paths from A to B, is $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$? Explain your answer.