MTH U341 Line Integrals To be turned in Wed., Dec. 3.

1. Let C be the curve $x^2 + y^2 = 4$, $x \geq 0$, $y \geq 0$. Suppose C represents the path followed by a particle P as P moves from the point $(2, 0)$ to the point $(0, 2)$ in the plane. The particle moves through a force field $\vec{F}$ given by $\vec{F}(x, y) = <x - y, x>$ kg. m/sec^2, and x and y are in meters.

a. Write down a line integral over C that computes the work W done by $\vec{F}$ on P.

b. Compute the integral by parametrizing C.

2. You are given the following data: $\vec{F}(x, y) = <P(x, y), Q(x, y)>$ is a vector field defined for $(x, y) \neq (0, 0)$ and the functions P and Q satisfy the equation

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$

In addition, you are given that

$$\int_{C'} P\,dx + Q\,dy = 5,$$

where C' is the unit circle $x^2 + y^2 = 1$, oriented in the counter-clockwise direction.

a. Let $D = \{(x, y) \text{ in } \mathbb{R}^2, (x, y) \neq (0, 0)\}$, that is, D is the plane with the origin $(0, 0)$ removed. Let A and B be points in D. Are line integrals $\int_{{}_A C_B} \vec{F} \cdot d\vec{r}$ independent of the choice of path C from A to B? If yes, explain why, if no, given an example of points A, B and two paths C_1, C_2 from A to B with $\int_{C_1} \vec{F} \cdot d\vec{r} \neq \int_{C_2} \vec{F} \cdot d\vec{r}$

b. Let $D = \{(x, y) \text{ in } \mathbb{R}^2, (x, y) \neq (0, 0)\}$, that is, D is the plane with the origin $(0, 0)$ removed. Is $\vec{F}$ conservative on D? Explain.

3. Now let $\vec{G}(x, y) = <P(x, y), Q(x, y)>$ be another vector field defined for all (x, y) in $\mathbb{R}^2$ and with

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$

(P and Q are different from what they were in #2).

a. Is $\vec{G}$ conservative on $\mathbb{R}^2$? Is there a potential function $g(x, y)$ for $\vec{G}$ on all of $\mathbb{R}^2$. Explain.

b. Let $D = \{(x, y) \text{ in } \mathbb{R}^2, (x, y) \neq (0, 0)\}$. Is $\vec{G}$ conservative on D? Explain.