MTH U341, Practice Final'08

Problem 1. Find the limit $\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$ if it exists or show that the limit does not exist.

Problem 2. Find the tangent plane to the surface

$$\vec{r}(u,v) = \langle u\sin(v), v\cos(u), u^2+v^2\rangle$$

at the point $(u,v) = (\pi/4, \pi/4)$.

Problem 3. Find the linear approximation of the function $f(x,y) = x^2y + \sqrt{xy}$ at the point $(1,2)$ and use it to approximate $f(1.5, 1.5)$.

Problem 4.

(1) Find the rate of change of $f(x,y) = ye^{x+y}$ at the point $(-1,1)$ in the direction of the vector $\langle 1,2\rangle$;

(2) Find the maximal rate of change of $f(x,y) = ye^{x+y}$ at the point $(-1,1)$ and the direction in which it occurs.

Problem 5. Find the local maximums, minimums, and the saddle points (if any) of the function $f(x,y) = x^3 + y^3 - xy - 2$.

Problem 6. Find the minimum and the maximum of the function $f(x,y) = x + y^2$ subject to the constraint $x^2 + y^2 = 1$.

Problem 7. Find the volume under the surface $z = xy$ over the region bounded by $y = 2x$, $y = \frac{1}{2}x$, and $x = 1$.

Problem 8. Evaluate the double integral $\iint_D y\,dA$ where D is the region bounded by $y = x^2$ and the line $x + y = 2$.

Problem 9. Find the mass of a lamina with density $\rho(x,y) = \sqrt{x^2+y^2}$ that lies in the first quadrant and is bounded by the circle $x^2 + y^2 = 4$ and the coordinate lines $x = 0$ and $y = 0$.

Problem 10. Evaluate the integral $\iiint\limits_E x\,dV$ where E is the solid tetrahedron bounded by the coordinate planes $x = 0$, $y = 0$, $z = 0$, and the plane $x+y+2z = 3$.

Problem 11. Evaluate the integral $\iiint\limits_E y\,dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

`Problem 12.` Evaluate the integral $\iiint\limits_E y\sqrt{x^2+y^2+z^2}\,dV$ where E is bounded by the xz-plane and the two hemispheres $\{x^2+y^2+z^2=1,\ y\geq 0\}$ and $\{x^2+y^2+z^2=4,\ y\geq 0\}$.

`Problem 13.` Let C be the straight-line path from the point $A=(2,0)$ to the point $B=(0,4)$. Let $\vec{F}$ be the force field $\vec{F}(x,y)=\langle x^2+2y, x-y^2\rangle$. Find the work done by $\vec{F}$ on a particle moving along C.

`Problem 14.` Consider the vector field, $\vec{F}(x,y)=\langle\sqrt{y}, \frac{x}{2\sqrt{y}}+2y\rangle$, $y>0$.

(1) Determine if $\vec{F}$ is conservative;

(2) Find a function $f(x,y)$ such that $\vec{F}=\nabla f$;

(3) Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$ where C is the straight interval connecting $A(1,1)$ with $B(3,2)$.

`Problem 15.` Use Green's theorem to evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=\langle e^x+y^2, e^{y^2}\rangle$ where C is the rectangle with corners $(2,2)$, $(-2,2)$, $(2,0)$ and $(-2,0)$, oriented counterclockwise.

`Problem 16.` Let S be the part of the paraboloid $z=1+x^2+y^2$ lying above the rectangle $0\leq x\leq 1$, $-1\leq y\leq 0$ and oriented by the upward normal. Compute the surface integral $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F}(x,y,z)=\langle xz, xy, yz\rangle$.

`Problem 17.` Use the Divergence theorem to evaluate the integral $\iint\limits_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F}=\langle x+\cos z, y, 2z+e^x y\rangle$ and S is the surface of the solid bounded by the planes $z=0$, $z=3$, and the cylinder $x^2+y^2=9$ (oriented by the outward normal).

`Problem 18.` Use Stokes' Theorem to calculate $\int_C \vec{F}\cdot dr$, where

$$\vec{F}=x^2z\,\vec{i}+xy^2\,\vec{j}+z^2\,\vec{k}$$

and C is the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=9$. (C is oriented counterclockwise when viewed from above.)