Review sheet for quiz 4

The quiz will cover §12.2, 12.3, 12.4, 12.5 and 12.7. Here is a list of topics that will be emphasized, some topics in the text that will NOT be on the quiz, and some suggested practice problems.

§12.1. Although this was covered on the last quiz, you will need to know and understand the definition of the definite integral of $f(x,y)$ over a rectangle as a limit of Riemann sums, and the interpretation of the double integral as a volume.

§12.2-Iterated integrals for rectangles. You should know:

- How to compute "partial integrals" $\int_a^b f(x,y)dx$ or $\int_c^d f(x,y)dy$.
- How to compute an iterated integral by first computing the "inside" partial integral, then integrating again to get a number.
- Fubini's theorem:

$$\int_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)dy\,dx = \int_c^d \int_a^b f(x,y)dx\,dy$$

  where R is the rectangle $[a,b] \times [c,d]$.
- How to use Fubini's theorem to compute volumes.

Besides the problems in the syllabus, practice problems: pg. 842-43 #13, 15, 17, 19, 21, 23, 25.

§12.3-Double integrals over general regions. You should know:

- What $\int\int_R f(x,y)dA$ means as a limit of Riemann sums, even if R is not a rectangle.
- How to compute $\int\int_R f(x,y)dA$ as an iterated integral with "variable" limits on the inner integral:

$$\int\int_R f(x,y)dA = \int_a^b \int_{B(x)}^{T(x)} f(x,y)dydx = \int_c^d \int_{L(y)}^{R(y)} f(x,y)dxdy.$$

- How to switch the order of integration. That is, change an iterated integral into a double integral over a region and then back into an iterated integral in the other order.

Besides the problems in the syllabus, practice problems: pg. 850-51 #7, 13, 15, 19, 25, 37, 39.

§12.4-Double integrals in polar coordinates. You should know how to change a double integral in rectangular coordinates into one in polar coordinates, using the transformation

$$x = r\cos\theta,\ y = r\sin\theta,\ dA = rdrd\theta,$$

and how to transform the given region of integration R in the x-y plane into the corresponding region R' in the r-θ plane.

Besides the problems in the syllabus, practice problems: pg. 856-75 #11, 19, 21, 25, 27.

§12.5-Applications of double integrals. You should know how to use double integrals for

- Finding volumes
- Finding mass, given the density
- Finding force, given the pressure
- Finding moments, moments of intertia and center of mass

You should also know the general principle of finding a quantity P for a region R if you are given the density Q of P per unit area as a function on R. You will not need to know the sections on probability or expected value.

Besides the problems in the syllabus, practice problems: pg. 866-67 #5, 9, 13, 17.

§12.7-Triple integrals. You should know how the whole story for double integrals works for triple integrals. This is

- the definition of the triple integral over a box as a limit of Riemann sums
- the extension of the triple integral to general solid regions
- the computation of a triple integral (either over a box or a more general region) by using a triple iterated integral: effectively, change the triple integral over a solid E to an integral of the form

$$\int\int\int_E f(x,y,z)dV = \int\int_R \left[\int_{B(x,y)}^{T(x,y)} f(x,y,z)dz\right] dA$$

  where R is the "shadow" of E in the x-y plane, and $B(x,y)$, $T(x,y)$ are the bottom and top of E when viewed from a point (x,y) in R.

You will not need to know how to apply triple integrals to computations of mass, volume moments, etc. (this will be on the next quiz).

Besides the problems in the syllabus, practice problems: pg. 879 #1, 9, 13, 25.

Additional practice problems: pg. 899-901(exercises) # 3,7,9,11,13,15, 17, 19, 21, 23, 25, 29, 31, 35, 41.