Review sheet for quiz 5

The quiz will cover §13.1, 13.2, 13.3 and 13.4. Here is a list of topics that will be emphasized, some topics in the text that will NOT be on the quiz, and some suggested practice problems.

§13.1-Vector fields. You should know

- What is a vector field, both given in terms of formulas and in terms of a picture.
- How to plot a vector field
- Examples of vector fields arising in physics and engineering: force field, velocity fields. Specific examples such as the gravitational field of a single mass and the electrical field of a point charge.
- Gradient fields and conservative vector fields.
- You should be aware of the potential functions for the gravitational and electrical fields

Besides the problems in the syllabus, practice problems: pg. 910, #9, 21

§13.2-Line integrals. You should know:

- There are two types of line integrals: line integrals with respect to arc length ds and "usual" line integrals, involving dx, dy, etc. You should know how these are defined as a Riemann sum, and how to apply this to e.g. computing arc length, mass or center of mass. You should know how to work with line integrals in the plane and in 3-space.
- How to compute a line integral by parametrizing the path of integration
- Line integrals of vector fields and the relation with work.

Besides the problems in the syllabus, practice problems: pg. 921-23 #7, 9, 17, 21, 28(just the mass, ans. $2r^2$), 35, 39, pg. 975, #3, 5

§13.3-The fundamental theorem. You should know:

- The statement of the fundamental theorem:

$$\int_{AC_B} \nabla f \cdot d\vec{r} = f(B) - f(A).$$

- How to explicitly find the potential function for a conservative vector field.
- How to use the fundamental theorem to simplify the computation of line integrals of a conservative vector field with a given potential function.
- What it means for a vector field to have path independent line integrals.
- That a vector field is conservative exactly when the vector field has path independent line integrals, and what this has to do with the fundamental theorem
- How to tell if a vector field $\vec{F} = \langle P(x,y), Q(x,y) \rangle$ defined on an open domain D in the plane is conservative, even without being able to find a potential function:

  If $\vec{F}$ is conservative, then $\partial P/\partial y = \partial Q/\partial x$.

  If $\partial P/\partial y = \partial Q/\partial x$ and D is simply connected, then $\vec{F}$ is conservative.

Besides the problems in the syllabus, practice problems: pg. 931-32 #7, 15, 17, 21, 29, 31, 33.

§13.4-Green's theorem. You should know

- what is a path connected region, and a simply connected region.
- what is the oriented boundary of a region.
- the statement of Green's theorem:
$$\iint_D \partial Q/\partial x - \partial P/\partial y \, dx\, dy = \int_{\partial D} P\, dx + Q\, dy.$$
and how to apply it in specific examples.
- How to use Green's theorem to simplify computations of certain path integrals, by changing the path to a simpler one.
- How to use Green's theorem to change an integral computing area of D to a line integral over ∂D:
$$\text{area}(D) = \iint_D 1\, dx\, dy = \int_{\partial D} P\, dx + Q\, dy$$
where $P(x,y), Q(x,y)$ are any convenient functions with $\partial Q/\partial x - \partial P/\partial y = 1$, for example
$$P = -(1/2)y,\ Q = (1/2)x$$

Besides the problems in the syllabus, practice problems: pg. 939-40#1, 7, 11, 15, 19, 27.

Additional practice problems: pg. 974(review) # 1-7. pg. 975-76(exercises)#1(a), 7, 9, 11, 13, 15, 17.