Review sheet for material after quiz 5: curl, divergence, surface integrals, Stokes' theorem and the divergence theorem

This sheet covers the material on §13.5, 13.6, 13.7 and 13.8 that we have covered since the last quiz; this material will be covered on the final exam. Here is a list of topics that will be emphasized, some topics in the text that will NOT be on the final, and some suggested practice problems.

§13.5-Curl and divergence. You should know

- How to compute curl and div of a vector field $\vec{F}$.
- The use of the "vector" $<\partial/\partial x, \partial/\partial y, \partial/\partial z>$ in defining/computing curl, div and grad.
- The relations satisfied by combining the operations curl, div and grad:

$$\text{div}(\text{curl }\vec{F}) = 0, \text{ curl (grad } f) = 0.$$

- The relation between curl = 0 and a vector field being conservative: If $\vec{F} = \nabla f$, then curl $\vec{F} = 0$. Conversely, if $\vec{F}$ is a vector field DEFINED ON ALL OF $\mathbb{R}^3$, and if curl $\vec{F} = 0$, then $\vec{F}$ is conservative (see theorem 13.5.4).

  Just as for vector fields in the plane, there are conservative vector fields on open regions D in $\mathbb{R}^3$, even if D is not all of $\mathbb{R}^3$, you just can't use the result quoted above to decide if a given vector field on such a D is conservative.
- Green's theorem in vector form: For R a bounded plane region, $C = \partial R$, $\vec{F}$ a vector field on R,

$$\int_C \vec{F} \cdot d\vec{r} = \int\int_R \text{curl}\vec{F} \cdot \vec{k}\, dA$$

  Here we consider $\vec{F}((x,y) = <P(x,y), Q(x,y)>$ as a vector field in $\mathbb{R}^3$ by setting

$$\vec{F}(x,y,z) = <P(x,y), Q(x,y), 0>.$$

Besides the problems in the syllabus, practice problems: pg. 946, #11, 15, 29.

§13.6-Surface integrals. You should know:

- That an oriented surface is a surface plus a continuous choice of a unit normal vector.
- There are two types of surface integrals: surface integrals of a function over an unoriented surface, respect to the scalar area dS: $\int\int_S f(x,y,z)\, dS$, and surface integrals over an oriented surface of a vector field, using dot product with $d\vec{S} = \vec{n}\, dS$:

$$\int\int_S \vec{F} \cdot d\vec{S} = \int\int_S \vec{F} \cdot \vec{n}\, dS.$$

  You should know how these are defined as a Riemann sum, and how to apply this to e.g. computing surface area or mass (for the first type) or for computing flux (for the second type).
- How to compute both types of surface integrals by parametrizing the surface. If the parametrization is given by $<x,y,z> = \vec{r}(u,v) = <x(u,v), y(u,v), z(u,v)>$,

you use the substitutions

$$dS = |\vec{r}_u(u,v) \times \vec{r}_v(u,v)|\, du\, dv; \quad \vec{S} = \vec{r}_u \times \vec{r}_v\, du\, dv.$$

- How to use the graph parametrization: if S is the graph of a function $f(x,y)$, you can take $\vec{r}(x,y) = <x, y, f(x,y)>$. Then

$$|\vec{r}_x \times \vec{r}_y| = \sqrt{f_x^2 + f_y^2 + 1}; \quad \vec{r}_x \times \vec{r}_y = <-f_x, -f_y, 1>.$$

- Other parametrizations, e.g., using cylindrical or spherical coordinates, when appropriate.

Besides the problems in the syllabus, practice problems: pg. 958 #9, 13, 21, 23, 25.

§13.7-Stokes' theorem. You should know:

- The statement of the Stokes' theorem and what each piece of the equation means: For S an oriented surface, $\vec{F}$ a vector field defined on S, we have

$$\int\int_S \text{curl } \vec{F} \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r}.$$

  This includes: what is curl $\vec{F}$, what is $d\vec{S}$, what is ∂S.
- How Stokes' theorem becomes Green's theorem if S is a plane region.
- How to use Stokes's theorem to (sometimes) simplify the computations of certain line integrals or surface integrals.

You do NOT need to know anything about the proof of Stokes' theorem. Besides the problems in the syllabus, practice problems: pg. 965 #9, 13, 17(don't do the line integrals, think of another way).

§13.8-The divergence theorem. You should know

- The statement of the divergence theorem: Let E be a solid region in $\mathbb{R}^3$ with boundary ∂E oriented by taking the normal that points away from E. Let $\vec{F}$ be a vector field on E. Then

$$\int\int\int_E \text{div } \vec{F}\, dV = \int\int_{\partial E} \vec{F} \cdot d\vec{S}.$$

- Examples of fields with divergence 0: $\vec{F}(\vec{x}) = (k/|\vec{x}|^3)\vec{x}$ (electrical field, gravitational field).
- How to apply the divergence theorem to simplify computations of flux integrals, especially for fields with 0 divergence (incompressible fields).

You do NOT need to know anything about the proof of the divergence theorem. Besides the problems in the syllabus, practice problems: pg. 971#5, 11, 15.

Additional practice problems: pg. 975-76(exercises)#19, 23, 25, 27, 31, 37.