Statement of Research

In a broad outline, my research interests lie in the areas of algebraic geometry, commutative algebra and representation theory. More precisely, in my dissertation, I studied the algebraic geometry of orbits of symmetric quiver representations. My own investigations and conversations with other people working in related fields has generated a number of interesting questions that relate the techniques of quiver theory to new results in algebraic geometry, Schubert calculus, representation theory and topology. These questions will drive my research for the next few years.

Though a relatively young concept in mathematics, representation theory of quivers grew quickly and in the last twenty years found applications in disparate areas of mathematics. A quiver is merely a directed graph while a quiver representation is a collection of vector spaces associated to each vertex of the graph along with linear maps associated to each arrow. At its very inception, the theory of quiver representations served to unify – and sometimes solve – many old problems in linear algebra. However, this new theory soon provided a language for non-commutative associative algebras, generated new problems in algebraic geometry and contributed to Schubert calculus. Today, a perusal of the current literature will reveal quivers playing key roles in the construction of canonical bases of universal enveloping algebras, a proof of the saturation of Littlewood-Richardson coefficients and the study of zero-dimensional Hilbert schemes.

Symmetric quivers extend the notion of a usual quiver by equipping it with a contravariant involution. In the standard theory, there exists a natural group action on any given quiver representation. The group acting on the affine representation space is always a subgroup of the general linear group on the direct sum of all vector spaces associated to each vertex of the quiver (the total vector space). However, when studying symmetric quivers, one defines symplectic or orthogonal representations in which the group action comes from a subgroup of the symplectic or orthogonal groups on the total vector space of the representation.

When, focusing on the algebraic aspects of the orbits of symplectic or orthogonal representations of symmetric quivers, I first provided a desingularization of the orbits of symmetric quivers of finite type. Using this, I obtained a codimension formula for the orbits of these representations in the appropriate representation space. Next, I attacked questions of normality and rationality of singularities and found that these orbits are not always normal. In the symmetric A_3 case, the orbit closure corresponds to natural symplectic or orthogonal analogues of the determinantal ideals. In that case, I also established minimal resolutions for (the normalization of) the coordinate ring of the orbit closure.

During the course of my work, the methods I used sometimes generated non-minimal resolutions for the coordinate ring of the orbit closures. However, when showing Cohen-Macaulay-ness, the resolutions I obtained hint at a new relationship between the dotted action of a Weyl group on the weight lattice of SL_n and the Littlewood-Richardson coefficients. This will likely constitute one of the next topics I investigate.

I also want to delve into the study of degeneracy loci; that is, cohomology classes of varieties defined by imposing rank conditions on a collection of vector bundles over a

scheme. At present, most articles concerning degeneracy loci emphasize algebraic geometry techniques, supplemented by the combinatorics of Schubert polynomials. Only a few articles in this field utilize tools from the topology of the associated linear algebraic groups and even fewer involve methods from the theory of quiver representations. Auslander-Reiten theory of quiver representations involves such nice combinatorics it should be possible to determine the coefficients of a degeneracy class in a cohomology ring in terms certain *Hom* and *Ext* spaces.

A third area of research that interests me involves extending the topological and algebraic theory of vector bundles to bundles of quiver representations. For example, using degeneracy loci as inspiration, I would like to investigate whether one can generalize Milnor's construction of characteristic classes of vector bundles from an infinite Grassmannian to a formulation that involves bundles of quiver representations. Some work has already been done in this area but I believe it still resides in inchoate form.

Finally, as a last example of topics that interest me, I mention briefly one that was born out of a course I taught entitled *Chaos & Fractals*. In presenting iterated functions to my students, I noticed that much of the work in the area of discrete dynamical systems uses topology and analytic function theory. Admittedly, this area is not much more than a recreational interest to me but as an algebraist I wondered if algebra might contribute new insights when one considers sequences within some field generated by iterating a polynomial function over a subfield. As a first result, I proved that the Galois group of the polynomial Φ_n whose roots are the n-cycles of a given polynomial iterated function is the subgroup of a certain wreath product. In particular, the order of $Gal(\Phi_n)$ is significantly smaller than $(\deg \Phi_n)!$, which a priori would be its upper bound did we know nothing about Φ_n except its degree.

One cannot predict precisely where one's results and developments in one's field will drive future research but the ideas presented above exemplify both the general and particular areas which I plan to study in the near future. I enjoyed my dissertation attacked because it intersected with many fields: algebraic geometry, algebraic groups, vector bundle theory, combinatorics and representation theory. My dissertation has taken me into many different areas of mathematics so from this starting point I hope to push further into various branches of algebra, algebraic geometry and representation theory, plumbing new ways in which these three areas interrelate.