Math 1137, Summer 2003
Homework 1: 1,3,8,23,24,29,33,34 p.15

Exercise: 1 p.15

a) "Boston is the capital of Massachusetts." is a proposition. Its truth value is T.
b) "Miami is the capital of Florida." is a proposition. Its truth value is F. (Talahassee is the capital.)
c) "2+3=5" is a proposition. Its truth value is T.
d) "5+7=10" is a proposition. Its truth value is F.
e) "$x+2=11$" is not a proposition. As written, we cannot decide whether it is true or not.
f) "Answer this question." is not a proposition.
g) "$x+y=y+x$ for every pair of real numbers x and y." is a proposition. Its truth value is T.

Exercise: 3 p.16

a) "Today is not Thursday."
b) "There is pollution in New Jersey."
c) "$2+1\neq 3$"
d) "It is not the case that the summer in Maine is hot and sunny." (Using what we will soon call DeMorgan laws, this can be rephrased as: "The summer in Maine is not hot or is not sunny.")

Exercise: 8 p.16
Le p, q and r be the following propositions: p="You have the flu." ; q="You miss the final exam."; r="You pass the course."
a) $p\rightarrow q$: "If you have the flu then you miss the final exam."
b) $\neg q\leftrightarrow r$: "You don't miss the final exam if and only if you pass the course."
c) $q\rightarrow\neg r$: "If you miss the final exam then you do not pass the course."
d) $p\vee q\vee r$: "You have the flu or you rmiss the final exam or you pass the course."
e) $(p\rightarrow\neg r)\vee(q\rightarrow\neg r)$: "Your having the flu implies that you don't pass the course or your missing the final exam implies you don't pass the course."
f) $(p\wedge q)\vee(\neg q\wedge r)$: "You have the flu and miss the final exam or you don't miss the final exam and you pass the course."

Exercise: 23 p.18
Truth tables for compound propositions.

a)

p	$\neg p$	$p\wedge\neg p$
T	F	F
F	T	F

b)

p	$\neg p$	$p\vee\neg p$
T	F	T
F	T	T

c)

p	q	$\neg q$	$p\vee\neg q$	$(p\vee\neg q)\rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d)

p	q	$p\vee q$	$p\wedge q$	$(p\vee q)\rightarrow(p\wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e)

p	q	①: $p\rightarrow q$	$\neg q$	$\neg p$	②: $\neg q\rightarrow\neg p$	①↔②
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

By the way, in the language of section 1.2 this is a proof that a conditional statement and its contropositive are logically equivalent.

f)

p	q	①: $p \to q$	②: $q \to p$	①→②
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Exercise: 24 p.18

Truth tables for compound propositions.

a)

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

b)

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

c)

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

In the language of section 1.2, this says that $(p \wedge q) \to (p \vee q)$ is a tautology, or in other words that $p \wedge q$ logically implies $p \vee q$ - also weritten $p \wedge q \Longrightarrow p \vee q$.

e)

p	q	$\neg p$	①: $q \to \neg p$	②: $p \leftrightarrow q$	①↔②
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f)

p	q	①: $p \leftrightarrow q$	$\neg q$	②: $p \leftrightarrow \neg q$	①⊕②
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Exercise: 29 p.18

Truth tables for compound propositions.

a)

p	q	r	$\neg q$	$\neg q \vee r$	$p \to (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b)

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

c)

p	q	r	$p \to q$	$\neg p$	$\neg p \to r$	$(p \to q) \lor (\neg p \to r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

d)

p	q	r	$p \to q$	$\neg p$	$\neg p \to r$	$(p \to q) \land (\neg p \to r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

e)

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \lor (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

f)

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Exercise: 33 p.18

a) $101\,1110 \quad OR \quad 010\,0001 = 111\,1111$
$101\,1110 \quad AND \quad 010\,0001 = 000\,0000$
$101\,1110 \quad XOR \quad 010\,0001 = 111\,1111$

b) $1111\,0000 \quad OR \quad 1010\,1010 = 1111\,1010$
$1111\,0000 \quad AND \quad 1010\,1010 = 1010\,0000$
$1111\,0000 \quad XOR \quad 1010\,1010 = 0101\,1010$

c) $00\,0111\,0001 \quad OR \quad 10\,0100\,1000 = 10\,0111\,1001$
$00\,0111\,0001 \quad AND \quad 10\,0100\,1000 = 00\,0100\,0000$
$00\,0111\,0001 \quad XOR \quad 10\,0100\,1000 = 10\,0011\,1001$

d) $11\,1111\,1111 \quad OR \quad 00\,0000\,0000 = 11\,1111\,1111$
$11\,1111\,1111 \quad OR \quad 00\,0000\,0000 = 00\,0000\,0000$
$11\,1111\,1111 \quad XOR \quad 00\,0000\,0000 = 11\,1111\,1111$

Exercise: 34 p.18

a) $1\,1000 \land (0\,1011 \lor 1\,1011) = 1\,1000 \land 1\,1011 = 1\,1000$

b) $(0\,1111 \wedge 1\,0101) \vee 0\,1000 = 0\,0101 \vee 0\,1000 = 0\,1101$

c) $(0\,1010 \oplus 1\,1011) \oplus 0\,1000 = 1\,0001 \oplus 0\,1000 = 1\,1001$

d) $(1\,1011 \vee 0\,1010) \wedge (1\,0001 \vee 1\,1011) = 1\,1011 \wedge 1\,1011 = 1\,1011$