Math 1137, Summer 2003

Homework 2: 6,7,13,19,29 p.26

Exercise: 6 p.26

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

The compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ have the same truth values so they are logically equivalent.

Exercise: 7 p.26

a) $(p \wedge q) \rightarrow p$ is a tautology:

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b) $p \rightarrow (p \vee q)$ is a tautology:

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c) $\neg p \rightarrow (p \rightarrow q)$ is a tautology:

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d) $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology:

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e) $\neg(p \rightarrow q) \rightarrow p$ is a tautology:

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f) $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology:

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Exercise: 13 p.26

We want to determine whether the following compound proposition is a tautology: $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$. We can do this in wo ways: either use a truh tbale or try to find equivalent logical propositions.

Truth table:

p	q	$p \to q$	$\neg q$	①:$\neg q \wedge (p \to q)$	②:$\neg p$	①→②
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

The truth table indicates that the compund proposition is in fact a tautology.

Second method: Using equivalent logical statements:

$$\begin{aligned}
(\neg q \wedge (p \to q)) \to \neg p &\iff (\neg q \wedge (\neg p \vee q)) \to \neg p && \text{by } p \to q \Leftrightarrow \neq p \vee q \\
&\iff ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \to \neg p && \text{by distributivity} \\
&\iff ((\neg q \wedge \neg p) \vee \mathbf{F}) \to \neg p \\
&\iff (\neg q \wedge \neg p) \to \neg p && \text{by identity} \\
&\iff \neg(\neg q \wedge \neg p) \vee \neg p && \text{by } p \to q \Leftrightarrow \neq p \vee q \\
&\iff (q \vee p) \vee \neg p && \text{by DeMorgan's law} \\
&\iff q \vee (p \vee \neg p) && \text{by associativity} \\
&\iff q \vee \mathbf{T} \\
&\iff \mathbf{T}
\end{aligned}$$

Exercise: 19 p.27

We wish to show that $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$. We'll make our life easy by simply doing a truth table.

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg p$	$\neg p \leftrightarrow q$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	T	F	T	F

We see that the fourth and sixth columns in the table have the same truth values. This proves that the two corresponding compound propositions are logically equivalent.

Exercise: 29 p. 20

The way the compound proposition is written is somewhat unclear since the author hasn't placed parantheses where they need to be. The exercise is to show that

$$①: ((p \to q) \wedge (q \to r)) \to (p \to r)$$

is a tautology.

p	q	r	②:$p \leftrightarrow q$	③:$(q \to r)$	④:②∧③	$p \to r$	①
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The last column of the truth table show that ① is a tautology. In otherwords, we could write that $(p \to q) \wedge (q \to r) \Longrightarrow p \to r$. In section 1.5, we will see that this is a rule of logical inference, called the "hypothetical syllogism".