<u>MTH U481: Summer 1 2005</u>

<u>Assignment 5</u>

<u>Due date:</u> Monday, June 13.

Reading: Chapter 7. (Estimation)

1). p. 277, #1.

2. A discrete random variable X takes the values $\{0, 1, 2\}$, with the following probabilities:

$$P(X = 0) = P(X = 1) = a, \qquad P(X = 2) = 1 - 2a$$

where a is an unknown parameter in the range $0 \leq a \leq 1$. Out of 100 measurements of X, the value 0 occurred 28 times, the value 1 occurred 32 times, and the value 2 occurred 40 times. Find the maximum likelihood estimator (MLE) $\widehat{a}$. [Hint: you can denote the number of orderings of the measurements by a constant C. Since it does not depend on a, its value is irrelevant to the answer]

3. The continuous random variable X has a pdf depending on one parameter θ. Suppose that $\widehat{\theta_1}$ and $\widehat{\theta_2}$ are independent estimators of θ, and both are unbiased. Suppose also that the mean square errors are

$$\mathrm{MSE}[\widehat{\theta_1}] = 12, \quad \mathrm{MSE}[\widehat{\theta_2}] = 20$$

Find the mean square error of the estimator

$$\widehat{\theta_3} = a\widehat{\theta_1} + (1 - a)\widehat{\theta_2}$$

and find the value of a which minimizes it.

4. A crude weighing scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = 0.1$. Suppose five successive weighings of the same object produce the results $3.142, 3.163, 3.155, 3.150, 3.141$.
a) Determine a 95% 2-sided confidence interval estimate of the true weight.

b) Determine a 99% 2-sided confidence interval estimate of the true weight.
c) Determine a 95% lower confidence interval estimate of the true weight.
d) Find the smallest number of additional weighings needed in order to determine the true weight to within ± 0.01 with 95% certainty.

5). p. 277, #9, #13.

6). p. 277, #14, #17, #27.

7). p. 277, #32.
Hints: for part (a), note that both X_{n+1} and $\overline{X}_n$ are normal, hence their difference is also normal. Find its mean and standard deviation. For part (b), use the fact on p.246 in Section 7.3.1, where it states that $\sqrt{n}(\overline{X}-\mu)/S$ is a t-random variable with $n-1$ degrees of freedom (we denoted this by T_{n-1} in class). Part (c) follows from part (b).

8). p. 277, #36.

9). p. 277, #42.