

## MTH U481: Summer 2005: Prof. M. Malioutov

### Assignment 3

Due date: Wednesday May 25.

Reading: Chapter 4. (Random variables)

1). A fair die is tossed three times. Let  $X$  be the number of different faces that appear (so  $X$  can have values  $\{1, 2, 3\}$ ). Find the pmf of  $X$ .

2). p. 131: #6, #7.

3). The pdf of a continuous random variable  $Y$  is given by

$$f(y) = \begin{cases} 0 & y < 0 \\ 2y & 0 \leq y \leq 1/2 \\ 6 - 6y & 1/2 < y \leq 1 \\ 0 & y > 1 \end{cases}$$

a) Sketch the graph of  $f(y)$ .

b) Compute the cdf  $F(y)$  (you should write the cdf as four different expressions in the four intervals given above).

c) Sketch the graph of  $F(y)$ .

d) Compute  $P(1/4 \leq Y \leq 3/4)$ .

4). p. 133: #22, #25, #27, #28.

5). Find the mean and variance of the discrete random variable  $X$  in Problem (1) above.

6). The pdf of a continuous random variable  $X$  is

$$f(x) = \begin{cases} \frac{3}{4} & 0 \leq x < 1 \\ \frac{1}{4} & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean, the variance, and the standard deviation of  $X$ .

7). Suppose  $X$  is a continuous random variable with pdf

$$f(x) = \frac{1 + kx}{2} \quad -1 \leq x \leq 1$$

where  $k$  is a constant between  $-1$  and  $+1$ . Compute  $E[X]$  and  $\text{VAR}[X]$  (your answers will depend on  $k$ ).

8. The numbers  $1, \dots, 4$  are written on four different cards, and the cards are placed in a box. Two cards are drawn at random without replacement. Let  $X$  and  $Y$  be the numbers on these cards.

- a). Find the joint pmf of  $X$  and  $Y$ .
- b). Find  $P(X + Y \leq 5)$ .
- c). Calculate the marginal pmf's of  $X$  and  $Y$ .
- d). Are  $X$  and  $Y$  independent? Explain.
- e). Calculate the mean and variance of  $X$ .
- f). Calculate the mean and variance of  $Y$ .
- g). Calculate the covariance of  $X$  and  $Y$ .

9. A point is chosen randomly and uniformly from the triangle with corners at  $(0, 0)$ ,  $(1, 0)$  and  $(0, 2)$ . Let  $X$  and  $Y$  be the coordinates of the point chosen.

- a). Find  $f(x, y)$ , the joint pdf of  $X$  and  $Y$ . Be sure to describe the region where  $f(x, y)$  is non-zero, and its value in that region.
- b). Calculate  $P(X \leq Y)$ .
- c). Calculate the marginal pdf's  $f_X$  and  $f_Y$  for  $X$  and  $Y$ . Again be sure to describe the intervals where they are non-zero.
- d). Are  $X$  and  $Y$  independent? Explain.
- e). Calculate the mean and variance of  $X$ .
- f). Calculate the mean and variance of  $Y$ .

10). p. 137: #45.