Thom polynomials for group actions

Global topology can *force* singularities. The following are examples. The topology of the Klein bottle forces self-intersections when mapped into 3-space. Any map of the projective plane into the plane must have (at least) cusp singularities. Any immersion of the projective plane into 3-space must have (at least) triple points. (We will see higher-dimensional examples involving more complicated singularities.) The topology of a manifold may force any differential form on it to degenerate at certain points. In a family of vector bundles over a complex curve some may be forced to degenerate to a non-stable bundle (in the GIT sense), depending on the topology of the family. In families of vector bundle maps—arranged according to a directed graph ('quiver')—some may be forced to degenerate, e.g., drop rank, etc., depending on the topology of the bundles involved. In families of linear subspaces some have special incidences with some other fixed ones (Schubert calculus).

There is a unified notion to describe these phenomena (due to Thom, Kirwan, Kazarian and others). It turns out that associated with a 'singularity' there is a universal characteristic class, that we call the Thom polynomial, which governs how that singularity is forced by global topology.

In this series of lectures we will review equivariant cohomology, define Thom polynomials and explore their geometric meaning. Then we study the following three problems.

- How to calculate the Thom polynomial of a given singularity. Different methods involve resolutions, interpolation, localization techniques, and Groebner bases.
- Applications in topology and geometry. Direct application computes the number (or cohomology class) of certain degenerations, incidences, or the degree of certain varieties, etc. In an indirect application, Thom polynomials serve as relations in cohomology rings of moduli spaces.
- The interior structure of natural infinite sequences of Thom polynomials (algebraic combinatorics). Combinatorial and interpolation properties of Schur, Schubert, and quiver polynomials will be derived.

Although the mainstream of the lectures is intended to be almost self-contained, knowledge of cohomology (i.e., singular cohomology of topological spaces) is needed, and familiarity with characteristic classes is helpful. Every now and then there will be digressions for those with more background in topology and/or algebra.