Corrections for Second Edition
Partial Differential Equations: Methods and Applications
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August 2005

p. 115-116. The proof of (iii) of the Proposition is incorrect. The correct proof is the following.

To prove (iii), let $v = \partial_j u = u_{(j)}$, then introduce

$$v_{(k)}(x) = \int_\Omega \partial_k \partial_j K(x-y)(f(y)-f(x))\,dy - f(x)\int_{\partial\Omega} \partial_j K(x-y)\nu_k\,dS_y.$$

(The domain integral converges because $f \in C^1$ implies $f(y)-f(x) = O(|y-x|)$ as $y \to x$.) We want to show that $v_{(k)} = \partial_k v$. Using the same function $\eta(t)$ as before, let us introduce the smooth function

$$v_\epsilon(x) = \int_\Omega \partial_j K(x-y)\,\eta\left(\frac{|x-y|}{\epsilon}\right) f(y)\,dy.$$

Clearly, $v_\epsilon \to v$ uniformly on compact subsets of $\mathbf{R}^n$ as $\epsilon \to 0$. For any fixed $x \in \Omega$, write

$$\partial_k v_\epsilon(x) = \int_\Omega \partial_k\left[\partial_j K(x-y)\,\eta\left(\frac{|x-y|}{\epsilon}\right)\right](f(y)-f(x))\,dy + f(x)\int_\Omega \partial_k\left[\partial_j K(x-y)\,\eta\left(\frac{|x-y|}{\epsilon}\right)\right]dy.$$

We can take $2\epsilon < \mathrm{dist}(x,\partial\Omega)$ to find

$$\int_\Omega \partial_k\left[\partial_j K(x-y)\,\eta\left(\frac{|x-y|}{\epsilon}\right)\right]dy = -\int_{\partial\Omega} \partial_j K(x-y)\nu_k\,dS_y,$$

where $\vec{\nu}$ is the exterior unit normal to Ω. Thus

$$v_{(k)}(x) - \partial_k v_\epsilon(x) = \int_{|x-y|<2\epsilon} \partial_k\left[\left(1-\eta\left(\frac{|x-y|}{\epsilon}\right)\right)\partial_j K(x-y)\right](f(y)-f(x))\,dy.$$

But, arguing as in the previous paragraph, we can show that

$$\int_{|x-y|<2\epsilon} \partial_k\left[\left(1-\eta\left(\frac{|x-y|}{\epsilon}\right)\right)\partial_j K(x-y)\right]|x-y|\,dy \le C(\epsilon),$$

where $C(\epsilon) \to 0$ as $\epsilon \to 0$, so

$$|v_{(k)}(x) - \partial_k v_\epsilon(x)| \le C(\epsilon)\sup_{y\in\Omega}\frac{|f(y)-f(x)|}{|y-x|}.$$

But $f \in C^1(\overline{\Omega})$ ensures that the supremum is finite and continuous in $x \in \Omega$, so we conclude that $\partial_k v_\epsilon \to v_{(k)}$ uniformly on compact neighborhoods of x as $\epsilon \to 0$. In particular, $v \in C^1(\Omega)$, and hence $u \in C^2(\Omega)$. ♠.

p. 254. In the last paragraph of the proof of Theorem 1, it is asserted that $\Delta(\phi u) = f'$ where $f' \in H^k(\mathbf{T}^n)$; in fact, this should be $f' \in L^2(\mathbf{T}^n)$. This effects the next step of the proof, since Theorem 2 of Section 8.1 now only yields $\phi u \in H^2(\mathbf{T}^n)$, which in turn implies $u \in H^2(\Omega')$. In order to obtain $u \in H^{k+2}(\Omega')$, this argument may be iterated, but some additional care must be taken because we always need to shrink the domain whenever we increase the regularity. Thus, in the first step, we should replace Ω' by Ω_1 and replace ϕ by the cutoff function $\phi_1 \equiv 1$ on Ω_1 to conclude $u \in H^2(\Omega_1)$. Iterating this argument yields $u \in H^3(\Omega_2)$, $u \in H^4(\Omega_3)$, etc. The condition $f \in H^k(\Omega)$ allows this iteration to continue until $u \in H^{k+2}(\Omega_{k+1})$ is obtained, and this implies $u \in H^{k+2}(\Omega')$.

p. 256. Formula (35b) is incorrect. The correct formula appears below.

$$\text{(35b)}\qquad \int_\Omega \phi\,\delta_j^{h_k} u\,dx = \int_\Omega (\delta_j^{-h_k}\phi)\,u\,dx \to -\int_\Omega \frac{\partial\phi}{\partial x_j}u\,dx.$$

p. 267. The first sentence of Exercise 1 is incorrect: $C = AB \geq 0$ is not necessarily true. The correct sentence reads as follows. Recall from linear algebra that if A and B are symmetric $n \times n$ matrices and $A, B \geq 0$, then the *trace* of $C = AB = (c_{ij})$ is nonnegative: $\mathrm{tr}(C) \equiv c_{11} + c_{22} + \cdots c_{nn} \geq 0$.

Note. *Thanks to Hal L. Smith of Arizona State University for bringing these errors to the author's attention.*