CALCULUS BC

Course Sequence Leading to Calculus

The majority of students enrolled in Calculus BC have been in Honors mathematics classes from grade 7. They studied Algebra 1 in grade 8, Geometry in grade 9, Algebra 2 in grade 10, and Pre-Calculus with Trigonometry in grade 11. Sometimes students who were new to Hingham or were otherwise not in the aforementioned sequence of courses prepare themselves for calculus by taking Geometry and Algebra 2 in grade 10. In any event all of the students have successfully completed four years of college preparatory mathematics, as described, above prior to grade 12. These students are well prepared to begin the study of Calculus.

Student Selection

Students receive recommendations for Calculus BC or Calculus AB from their grade 11 teacher. Students and their parents decide which, if either, of these 2 courses they will take in grade 12. They do not need to be recommended for Calculus BC to enroll in the course. Usually there are two sections of Calculus AB and one section of Calculus BC. The Calculus BC is taught during the same class period as one of the Calculus AB classes to facilitate movement from Calculus BC to Calculus AB for students that find the BC course too difficult. The experience has been that the BC class is very stable. In a typical year about 15 students start and complete Calculus BC. All students in Calculus BC take the AP Examination.

Calculus BC Course Outline

This outline is based on 150 school days prior to the AP Examination. The number of days include days for testing and review.

Unit I. Limits & Continuity - 6 Days - 1 Test

1. From graphs
2. From tables
3. Symbolic evaluations
4. Limits at infinity
5. Infinite limits
6. Indeterminate forms: $\frac{0}{0},\ \frac{\infty}{\infty},\ \infty-\infty,\ 0\cdot\infty$
7. Graphical look at removable discontinuities
8. Graphical look at non-removable discontinuities
9. Symbolic consideration of removable discontinuities
10. Symbolic consideration of non-removable discontinuities

Unit II. Derivatives - 21 Days - 1 Test

1. Average rate of change - related to velocity
2. Average rate of change - related to slope
3. Instantaneous velocity as the average velocity over a smaller time interval
4. Instantaneous velocity as the slope of a curve at a point
5. Local linearity
6. Definition of the derivative as a limit
7. Approximate the derivative at a point graphically
8. Approximate the derivative at a point numerically
9. Determine the graph of the derivative function from the graph of a function
10. Determine the derivative of a function by using the limit definition
11. Explore the relationship between differentiability and continuity
12. Practical meaning of the derivative in a variety of contexts
13. Techniques of differentiation - Power rule, Product rule, Quotient rule
14. Chain rule
    (a) using Leibniz notation
    (b) using function notation
    (c) using parametric equations
15. Implicit functions
16. Inverse functions - using composition - eg. use $e^{\ln x} = x$ to obtain $d(\ln x)/dx$
17. Graphical meaning of the second derivative
18. Key theorems relating to continuous functions
    (a) Mean Value Theorem
    (b) Intermediate Value Theorem
    (c) Extreme Value Theorem

Unit III. Applications of the Derivative - 21 days - 1 Test

1. Approximations using the tangent line
2. Related rates
3. Intervals of increase and decrease of a function
4. Intervals of increase and decrease of the derivative - concave up and concave down
5. First derivative test
6. Second derivative test
7. Candidates test
8. Optimization
9. Geometric view of a solution to a differential equation using slopefields
10. Euler's Method to approximate the solution to a differential equation
11. L'Hopital's Rule for cases of $0/0$ and ∞/∞
12. Motion on a line - moving left and right, speeding up and slowing down
13. Relationship of moving right and speeding up to a graph that is increasing and concave up, moving left and slowing down to decreasing and concave up etc.

Unit IV. Integration and Antidifferentiation - 6 Days - 1 Test

1. Variety of examples of summing to approximate total change given tabular data.
2. Concept of a Riemann sum
3. Definite Integral defined as the limit of a Riemann sum
4. Link between the definite integral and area - advantages and pitfalls
5. Properties of the definite integral
6. Antidifferentiation motivated by finding a position function from a velocity function.
7. Fundamental Theorem of Calculus motivated by finding distance traveled 2 different ways

Unit V. Numerical Approximations of a Definite Integral - 5 Days - No Test

1. Riemann Sums - Left, Right, Midpoint
2. Trapezoid Rule
3. Simpson's Rule
4. Relationship between Trapezoid, Midpoint, and Simpson's Rules
5. Investigation as to how each of these techniques improves if the number of subdivisions is doubled, tripled, multiplied by a factor of k

Unit VI. Techniques of Antidifferentiation - 11 days - 1 Test

1. From known derivatives
2. From a graph of a derivative
3. Simple substitution - form completion
4. Substitution - actual substitution needs to be made, including trig substitution
5. Parts
6. Improper Integrals

Unit VII. Applications of Definite Integral and Antidifferentiation - 32 Days - 2 Tests

1. Determine specific antiderivatives using initial conditions
2. Solution to separable differential equations with and without initial conditions
3. Writing a differential equation to translate a verbal description
4. Partial fractions in the context of the logistic equation
5. Representation of a particular antiderivative by using The Fundamental Theorem of Calculus
6. Analysis of functions defined by a definite integral
7. Area including regions bounded by polar curves
8. Average value of a function
9. Distance as the definite integral of speed
10. Length of a curve including polar and parametric curves
11. Work
12. Variety of other problems using the integral of a rate of change to determine total or accumulated change
13. Variety of other problems where the emphasis is on setting up a Riemann sum and taking its limit

Unit VIII. Series - 38 Days - 3 Tests.

1. Infinite series defined as the limit of a sequence of partial sums
2. Series of constants
   (a) Geometric Series
   (b) Harmonic Series, P-Series
   (c) Alternating Series
3. Tests for convergence
   (a) Integral
   (b) Comparison
   (c) Limit Comparison
   (d) Ratio Test - thought of as eventually geometric
4. Power Series
   (a) Taylor Polynomials as approximations for functions
   (b) Taylor series centered at x = a
   (c) Use of known Maclaurin series for e^x, $\sin x$, $1/(x+1)$, $(1+x)^p$ to form new series
   (d) Differentiation and antidifferentiation of series to determine new series
   (e) Functions defined by power series
   (f) Interval and radius of convergence
   (g) Error bounds
      (i) convergent geometric series
      (ii) using integral test

(iii) convergent alternating series
(iv) Lagrange

Review For AP Exam - 10 Days

References and Materials

Major Texts

Deborah Hughes-Hallett, Andrew M. Gleason, McCallum et al. Calculus, fourth edition. New York: John Wiley & Sons 2005.

Howard Anton. Calculus. Third edition / Brief edition. New York: John Wiley & Sons 1988.

Finney, Demana, Waits, Kennedy. Calculus – Graphical, Numerical, Algebraic, second edition. Addison Wesley Longman 1999.

Reference Books

Thomas P. Dick, Charles M. Patton. Calculus of a Single Variable. Boston: PWS Publishing Co. 1994.

Arnold Ostebee, Paul Zorn. Calculus from Graphical, Numerical, and Symbolic Points of View. Harcourt College Publishers 1997.

George Best, Stephen Carter, Douglas Crabtree. Concepts and Calulators in Calculus. Andover, MA: Venture Publishing 2000

North Carolina School of Science and Mathematics; Contemporary Calculus through applications; Dedham, MA: Janson Publications; ISBN 0-939-765-87-X

Resources for Calculus Collection, Volume 1: Learning by Discovery: A Lab Manual for calculus, Anita Solow, Editor; ISBN 0-88385-083-4

Resources for Calculus Collection, Volume 2: Calculus Problems of the New Century, Robert Fraga, Editor; ISBN 0-88385-084-2

Resources for Calculus Collection, Volume 3: Applications of Calculus, Phillip Straffin, Editor; ISBN 0-88385-085-0

Resources for Calculus Collection, Volume 4: Problems for Student Investigation, Michael Jackson and John Ramsay, Editors; ISBN 0-88385-086-9

Resources for Calculus Collection, Volume 5: Readings for Calculus, Underwood Dudley, Editor; ISBN 0-88385-087-7

Calculus: An Active Approach With Projects; The Ithaca College Group; John Wiley & Sons; ISBN 0471-00316-6

George Best, Sally Fischbeck. AP Calculus with the TI-83 Graphics Calculator. Andover, MA: Venture Publishing

George Best, J. Richard Lux. Preparing for the AP Calculus(BC) Exam. Andover, MA: Venture Publishing

Judith Broadwin, George Lenchner. Solutions to AP Calculus Problems Part II AB and BC. MOES.

Sources For Calculator Programs

George Best, Sally Fischbeck. AP Calculus with the TI-83 Graphics Calculator. Andover, MA: Venture Publishing

Deborah Hughes-Hallett, Andrew M. Gleason, et al. Instructors Manual With Sample Exams To Accompany Calculus, second edition. New York: John Wiley & Sons 1998.

Dennis D. Berkey, Paul Blanchard. Calculus - third edition. Saunders College Publishing; ISBN 0-03-046927-9

Teaching Strategies

1. Each topic is presented numerically, geometrically, symbolically, and verbally.
2. The students are encouraged to express their ideas in carefully written sentences.
3. The students make extensive use of the TI-83 calculator. Each student has his or her own calculator.
4. The students use programs in their calculators to:
   (a) Perform numerical integration
   (b) Find points of inflection
   (c) Show Riemann Sums
   (d) Compute Partial Sums
   (e) Use Euler's Method
   (f) Show a slopefield
   (g) Draw a solution curve on a slopefield
   (h) To sketch implicitly defined functions
5. From the middle of October throughout the year the students are assigned 3 free response questions from old AP Exams every three days. These questions are graded as they would be at an AP reading. Students may use a calculator for any question for which a calculator was allowed when the question appeared on an AP Exam and they may not use a calculator for any question for which a calculator was not allowed when the question appeared on an AP Exam.
6. Each week they have one or more surprise quizzes containing 5 multiple choice items from released AP exams. Calculator usage is the same as described above.
7. All tests contain material from previous units. The students are responsible for all material covered to the date of a test. All tests are 2 periods in length, one with calculator usage and one without.
8. The students are encouraged to work cooperatively on in-class worksheets, graded AP problems, take-home exams.
9. Circular functions, exponential functions, logarithmic functions are used throughout the course. The students have previously studied these functions so we deal with the derivatives of these functions early in the course.
10. The students learn to use the spreadsheet program Excel.
11. Excel is used for Euler's method and for summing examples.

Remarks

My advice to teachers who are beginning a new AP Calculus course or who are unsure of how best to deal with the topics in the Course Description is that they attend a summer institute that will deal with this Course Description as well as how to integrate technology into the course.

It is not essential to adopt one of the newer calculus textbooks as the book for the course, however, you will need to use material from these texts or develop your own to successfully accomplish the course. I am currently using a traditional and a "reform" text throughout the course. Some topics are taught using only one of the texts, but most are taught using both. Additionally, I use a third text for the study of Series.