Do not write in this space:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ

Instructor:___________________________ Name: ________________________________

NORTHEASTERN UNIVERSITY

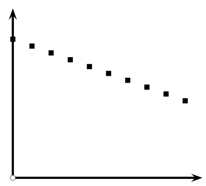
Department of Mathematics

MTH U131 (Calculus for Business and Economics) Final Exam Fall 2006

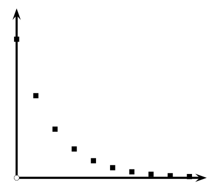
Instructions: Put your name and your instructor's name in the blanks above. Put your final answers to each question in the designated spaces on these test pages. (You may lose all credit for a problem if you don't.) **SHOW YOUR WORK.** If there is not enough room to show your work, use the back of the preceding page. **Whenever you use nDeriv or fnInt on your calculator, say so.** Always tell what function is in Y_1, Y_2, etc. in your calculator. For your convenience, there is a table of formulas at the end of the exam. On this exam, you may only use one of the following calculators: **TI-83, TI-83$^+$, TI-84 or TI-84+**. Total Points = 400

1. For each scatter plot below, tell whether it is best described as: Linear, Exponential, Quadratic, Cubic, Logistic or None. The same answer might be used more than once. (5 points each)

a.



b.



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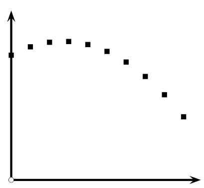

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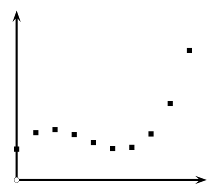


2. The revenue of a company, in millions of dollars, t years after 1990 is given in the following table

Years since 1990	2	4	6	8	10	12	14
Revenue in millions of dollars	10	20	60	90	110	115	118

a) (16 points) Let $R(t)$ denote the revenue t years after 1990. Use the table above to fit the best model for $R(t)$, **among the following choices: LINEAR, QUADRATIC, CUBIC, EXPONENTIAL and LOGISTIC**. Write the formula for $R(t)$ here rounding each coefficient of $R(t)$ to 3 decimal places.

b) (8 points) Use the model in part (a) to find the average rate of change of revenue between 1993 and 1997. Show work and give your answer (rounded to two decimal places) with units. If you use the exact model say so.

3. Find the derivative of each of the following functions (17 points each).

a) $g(x) = 6(3.41)^x + 10\sqrt{x^7} - 8x^{-2} - \dfrac{\ln(x)}{4}$

b) $m(x) = \ln(12^{-x} + 11) - \dfrac{2.75}{x^4}$

c) $f(x) = \dfrac{50}{1 + 9e^{0.02x}}$

d) $h(x) = 5(2^x)(7x^6 - 3e^x)$

4. Let $N(x)$ be the number of video game consoles sold, in millions, if the price of each console is x hundred dollars. In parts a), b), and c), of this problem circle the number of the correct answer. **There is only one correct answer and there is no partial credit.**

a) (6 points) The practical meaning of $N(5) = 4$ is:

(i) When the price is \$500, 4 million consoles are sold.

(ii) When the price is \$400, 5 million consoles are sold.

(iii) When the price is \$500, the revenue is \$ 4 million.

(iv) When the price is \$400, the revenue is \$ 5 million dollars.

(v) None of the above

b) (6 points) Give the units of $N'(x)$:

(i) dollars per console

(ii) hundreds of consoles per million dollars

(iii) millions of consoles per hundred dollars

(iv) consoles per dollar

(v) None of the above

c) (12 points) If $N(5) = 4$ and $N'(5) = -0.6$, estimate how many consoles will be sold if the price of a console is \$550.

(i) 4.6 million (ii) 3.7 million (iii) 4.3 million (iv) 3.4 million (v) None of the above

5. A department store chain sells greeting cards.

a) (12 points) During the holiday season, the profit (in dollars) of the chain from selling greeting cards is

$$P(x) = -0.6x^3 + 5.4x^2 + 198.8x - 100,$$

where x is the number of thousands of cards sold. Write the formula for the **MARGINAL PROFIT** function. Include the unit in your answer.

b) (12 points) The chain's **MARGINAL PROFIT** from selling 8000 cards is \$0.17 per card. The practical meaning of this fact is: (Circle the number of the correct answer. There is only one, and there is no partial credit.)

(i) The profit from selling 8000 cards is exactly \$1360.

(ii) The average profit from selling one card is about \$0.17.

(iii) The profit from selling 8000 cards is about \$1360.

(iv) The profit from selling the 8001^{st} card is about \$0.17.

(v) None of the above

6. A shoe store's profit function for selling running shoes during its annual sale is

$$P(x) = (0.01x^3 - 50)(0.914)^x \quad ,$$

where $P(x)$ is the profit in **hundreds of dollars** when the selling price of each running shoe is x **dollars**. Carefully enter the function $P(x)$ into your calculator. **Check that you have entered the function correctly by computing $P(5)$. You should obtain -31.09607669.**

a) (20 points) From past experience, the owner of the store knows that she shouldn't sell a running shoe for more than \$50. Use *nDeriv* and your calculator to determine the price that maximizes PROFIT. Show your work **especially the equation that you must solve and tell how you use the calculator** by filling in the blanks below. Write the calculator answer with ALL the decimal places the calculator gives.

equation to solve: ______________________________

calculator procedure: ______________________________

calculator answer: ______________________

b) (20 points) Use a derivative test to show that the price you found in part (a) maximizes the profit.

c) (5 points)(i) Round off the answer in part (a) to the nearest cent: Answer: __________.

(ii) Use your answer in (i) to find the maximum profit. Round the value of the maxiumum profit to two decimal places and include the units. Show work.

Maximum Profit: ______________________________.

7. (18 points) *Al's Bakery* sells birthday cakes. Based on a marketing poll, Al, the shop's owner, has found the following model for the demand function (i.e., the number of birthday cakes sold each week):

$$D(x) = 5 + \frac{600}{1 + .05x^2} \quad ,$$

where the price of each cake is x dollars. Each cake costs the shop \$8.50 to make. The shop has fixed costs of \$35 for the advertising display for the cakes. Write down the formulas for $R(x)$, $C(x)$, and $P(x)$, the revenue, cost, and profit functions (in dollars). **Write each formula out in full. Do not use any abbreviations.**

R(x) = ____

C(x) = ____

P(x) = ____

8. Read this problem carefully. **Each part of this problem is about a DIFFERENT function.**

a) (8 points) If $f'(x) = (8x - 9)(x + 3)$, what are the critical points of $f(x)$, i.e., give the x-coordinate of each critical point of $f(x)$. USE ALGEBRA, not special features of the calculator. Show your work especially the equation you must solve and how you solve it. (Note: You are given the **DERIVATIVE** of $f(x)$.)

b) (16 points) Suppose $g(x)$ has critical points at $x = 1$ and at $x = 4$. If $g'(x) = (x - 1)(x - 4)^2$, circle the correct statement for each critical point. Show work. (Note: You are given the **DERIVATIVE** of $g(x)$.)

(i) $x = 1$ is a relative max relative min neither

WORK:

(ii) $x = 4$ is a relative max relative min neither

WORK:

c) (20 points) (i) **Use derivatives and algebra** to find the inflection point of the function $h(x) = 2x^3 - 12x^2 + 50$. Give its x and y coordinates. Show your work especially the equation you must solve and how you solve it.

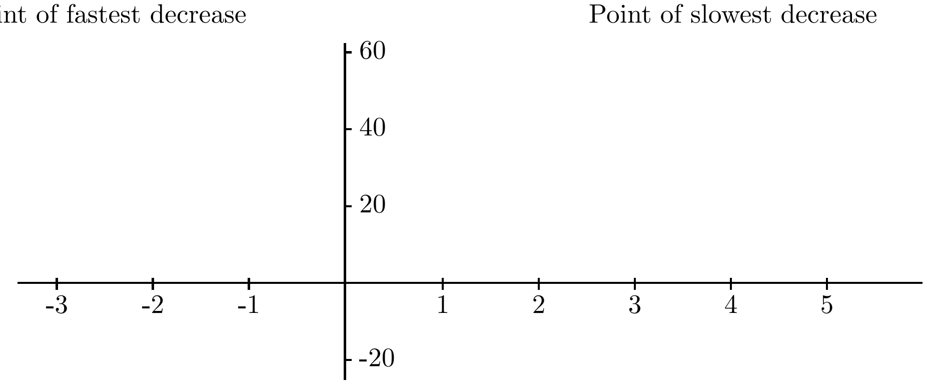
(ii) Graph $h(x)$ (from part (i)) using the window: $X_{min} = -2$, $X_{max} = 5$, $Y_{min} = -20$, $Y_{max} = 60$. Sketch what you see below. In your sketch, **label the inflection point clearly with its coordinates**. Use the graph to decide what kind of inflection point the graph has. Circle one of the following.

Point of fastest increase

Point of slowest increase

Point of fastest decrease

Point of slowest decrease



9. (12 points) Evaluate the integral, i.e., find the general antiderivative.

$$\int (7x^{12} - 2e^{3x} - \frac{1}{5})\ dx$$

10. In parts a), b), c), and d) circle the number of the correct answer. (8 points each, no partial credit).

a) An antiderivative of $g(x) = \dfrac{1}{4\sqrt{x}}$ is:

(i) $\ln(4\sqrt{x})$ (ii) $\dfrac{x^{1/2}}{2}$ (iii) $\dfrac{\ln(|x^{1/2}|)}{4}$ (iv) $\dfrac{x^{3/2}}{6}$ (v) None of the above

b) An antiderivative of $h(x) = 4(2.5)^x$ is:

(i) $\dfrac{4(2.5)^x}{\ln(2.5)}$ (ii) $\dfrac{10^x}{\ln(10)}$ (iii) $4(2.5)^x \ln(2.5)$ (iv) $10^x \ln(10)$ (v) None of the above

c) An antiderivative of $k(x) = \dfrac{5}{2x}$ is:

(i) $\dfrac{-5}{2x^2}$ (ii) $-\dfrac{5}{2}x^0$ (iii) $\dfrac{5}{2}\ln(|x|)$ (iv) $5\ln(|2x|)$ (v) None of the above

d) An antiderivative of $m(x) = \dfrac{d(e^{x^2})}{dx}$ is:

(i) e^{x^2} (ii) $2xe^{x^2}$ (iii) $\dfrac{e^{x^2}}{x^2}$ (iv) $\dfrac{e^{x^2}}{\ln(x^2)}$ (v) None of the above

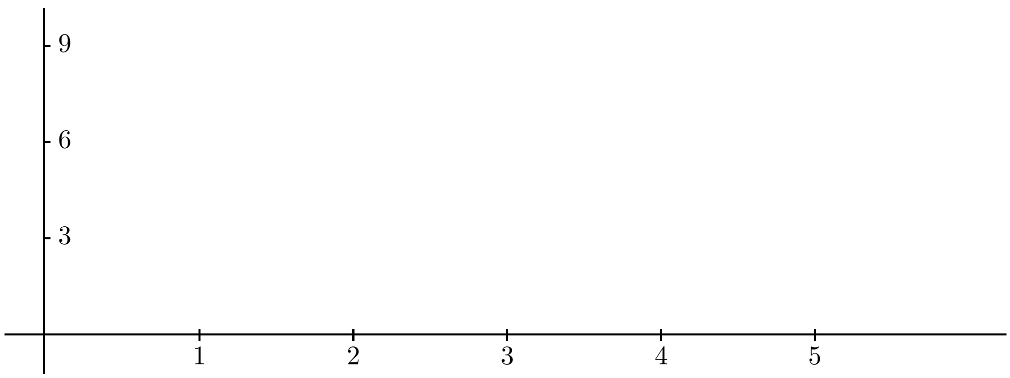
11. (17 points) Evaluate the following integral. Show all work. *Use your calculator only to perform basic arithmetic.* Do not use $fnInt$. **A numerical answer without appropriate work shown will receive no credit**.

$$\int_1^8 (2x^{1/3} - 8x^{-2})\ dx$$

12a. (8 points) Make a careful sketch of the region whose area is given by the definite integral:

$$\int_{1}^{5} \sqrt{3x^2 + 6}\, dx.$$

Label the boundary curves (with their equations) and corner points (with their coordinates) of the region. Then shade the region.



12b. (12 points) When rounded to 2 decimal places, the 2 left rectangle approximation (in square units) of the area in part (a) is: (Circle the number of the correct answer)

(i) 23.28 (ii) 17.49 (iii) 22.98 (iv) 16.97 (v) None of the above

13. (16 points) Find $F(x)$, the specific antiderivative of the function $f(x)$, where $f(x) = 14x^6 - 8x$ and $F(-1) = 13$. Show all your work especially **any equations you solve**.

14. The balance (in dollars) in an investment account t years after January 1, 2000 is given by the formula:

$$A(t) = 1500e^{0.06t}.$$

a) (6 points) Find the rate of change of the balance in the account on January 1, 2003. Show work. Give your answer (rounded to two decimal places) with units.

b) (18 points) Find the average balance in the account between January 1, 2003 and January 1, 2006. Show your work, including the **formula for the average balance** and how you use any special features of the calculator. Give your answer (rounded to two decimal places) with units.

15. (12 points) Let $p(t)$ be the rate of change of the number of cellular telephone subscribers (in the U.S.), in thousands of subcribers per year, t years after 1988. What is the practical meaning of the following integral?

$$\int_2^6 p(t)\ dt$$

Circle the number of the correct answer. (There is only one, and there is no partial credit.)

(i) The average number of subscribers in thousands from 1990 to 1994.

(ii) The change in the number of subscribers from 1990 to 1994 in thousands

(iii) The average rate of change of the number of subscribers between 1990 and 1994 in thousands of subscribers per year

(iv) The sum of the number of subscribers in 1990 and the number of subscribers in 1994 in thousands

(v) None of the above

List of Formulas

Derivatives: $(x^n)' = nx^{n-1}$, $(e^x)' = e^x$, $(a^x)' = (\ln\ a)a^x$, $(\ln x)' = \dfrac{1}{x}$;
(chain rule) $(f(g(x)))' = f'(g(x))g'(x)$; (product rule) $(fg)' = fg' + f'g$

Approximation Formula for the change in a function using the derivative:

$$f(x+h) - f(x) \approx f'(x)h$$

Antiderivatives: $\displaystyle\int e^{bx}\ dx = \frac{e^{bx}}{b} + C$, $\displaystyle\int \frac{1}{x}\ dx = \ln(|x|) + C$, $\displaystyle\int x^n\ dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$, $\displaystyle\int a^x\ dx = \frac{a^x}{\ln(a)} + C$.

Fundamental Theorem of Calculus: If $\dfrac{dF}{dx} = f(x)$, then $\displaystyle\int_a^b f(x)\ dx = F(b) - F(a)$.

The average value of $f(x)$ over the interval from x=a to x= b is given by the following expression:

$$\frac{1}{b-a}\int_a^b f(x)\ dx.$$

Consumer Surplus: Let $D(x)$ be the number of items sold at price x dollars. The consumer surplus when the market price of the item is p_0 dollars, and p_1 dollars is the highest price that anyone will pay for the item, is given by the following expression.

$$\int_{p_0}^{p_1} D(x)\ dx.$$