Solutions to MTH 4131 Review Exercises for F

1a) Logistic   b) Linear

c) None   d) Quadratic

Note on c): A quartic model would be appropri
but it is not one of the choices

2a) There seems to be a change in concavity near the data point (9, 18) so a cubic model seems appropriate.
So using Cubic Reg Y_1, we find that

$$V(x) = .1435x^3 - 2.7391x^2 + 18.4993x - 31.2371$$

b) If Y_1 is the full model, then the average rate of change $= \dfrac{Y_1(8) - Y_1(4)}{8-4} = 1.7044 \dfrac{\text{billion \$}}{\text{year}}$

c) instantaneous rate of change
$\approx$ nDeriv $(Y_1, X, 11) = 10.3366 \dfrac{\text{billion \$}}{\text{year}}$

3a) $f(x) = \frac{2}{5}x^{-2} + 3x^7 - 3x^{0.5}$

$f'(x) = -\frac{4}{5}x^{-3} + 21x^6 - 1.5x^{-0.5}$

b) $g(x) = 3x^{1/5} + 4x^{-10} + 2^x$

$g'(x) = \frac{3}{5}x^{-4/5} - 40x^{-11} + 2^x \ln(2)$

3c) $j'(x) = 5\cdot(3e^x + 3x^2)\cdot 14(3e^x + x^3)^{13}$

d) $h'(x) = 990(40x^3 - 3)\, e^{(10x^4 - 3x)}$

e) $f'(x) = 2401(0.08)(0.6\, e^{0.6x})(-(1+.08e^{0.6x})^{-2})$

f) Since $p(x) = 4x^{2/3} + 350(1.04)^x$

$p'(x) = \frac{8}{3}x^{-1/3} + 350 \ln(1.04)(1.04)^x$

4. Apply the product rule to conclude that the correct answer is (d)

5. $R(x) = -7.03x^2 + 166.6x + 47.13$

(a) $R'(x) = -14.06x + 166.6$ \$/hundred hotdogs

(b) $R'(3) = 124.42$ \$/hundred hotdogs

(c) $R(3) = \$483.66$

(d) $R(3.2) \approx R(3) + 0.2\,R'(3)$

$= 483.66 + 0.2(124.42)$

$= 483.66 + 24.88 = \$508.54$

6 a) $\int (10(8^x) + 3e^{-4x})\,dx$

$$= \frac{10(8^x)}{\ln 8} + \frac{3e^{-4x}}{-4} + C$$

b) $\int \left(\frac{12}{7x^5} - \frac{7e^x}{9}\right)dx = \int \left(\frac{12}{7}x^{-5} - \frac{7}{9}e^x\right)dx$

$$= \frac{12}{7}\frac{x^{-4}}{-4} - \frac{7}{9}e^x + C$$

or $-\frac{3}{7}x^{-4} - \frac{7}{9}e^x + C$

c) $\int \left(\frac{65.7}{x} - 4\sqrt[3]{x}\right)dx = \int \frac{65.7}{x} - 4x^{1/3}\,dx$

$$= 65.7\ln|x| - 4\frac{x^{4/3}}{4/3} + C$$

or $65.7\ln|x| - 3x^{4/3} + C$

d) $\int (21x^6 - 2.4x^7 - 10^2)\,dx$

$$= 21\frac{x^7}{7} - \frac{2.4x^8}{8} - 100x + C$$

or $3x^7 - 0.3x^8 - 100x + C$

Solutions to Final Exam Review Exercises

7.

$$R(x) = x \cdot D(x) = \frac{100\,000\, x e^{-.81x}}{1+50e^{-.81x}}$$

$$C(x) = 1.75D(x) + 1600 = \frac{175000\, e^{-.81x}}{1+50e^{-.81x}} + 1600$$

$$P(x) = R(x) - C(x) = \frac{100000\, x e^{-.81x}}{1+50e^{-0.81x}} - \frac{175000 e^{-.81x}}{1+50e^{-.81x}} - 1600$$

8. $P(x) = (100x^2 - 500)(1+5e^{0.4x})^{-1}$

a. Solve $n\text{Deriv}(P(x), x, x) = 0$

by graphing $n\text{Deriv}(P(x), x, x)$ on the interval $0 \le x \le 12$ and using 2nd calc zero to find where $n\text{Deriv}(P(x), x, x)$ crosses the x-axis.

the calculator answer is: 5.9354814

b. (i) Comparing function values

$P(5.0) \approx 52.707 < P(5.9354814) \approx 55.253$

$P(6.0) \approx 55.243 < P(5.9354814) \approx 55.253$

(ii) first derivative test

$n\text{Deriv}(P(x), x, 5) \approx 5.826 > 0$, $n\text{Deriv}(P(x), x, 6) \approx -.319 < 0$

(iii) 2nd derivative test

Let $Y_2 = n\text{Deriv}(P(x), x, x)$, then $n\text{Deriv}(Y_2, x, 5.9354814) \approx -5.023 < 0$

c. Rounded price giving maximum profit: \$5.94

Maximum profit: $P(5.94) \approx 55.25$ thousand dollars

Solutions to Final Exam Review Problems

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9a. Solve $f'(x) = -6x^2 + 6x + 72 = 0$

$$\frac{-6x^2}{-6} + \frac{6x}{-6} + \frac{72}{-6} = \frac{0}{-6}$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x - 4 = 0 \Rightarrow x = 4; \quad x + 3 = 0 \Rightarrow x = -3$$

b. (i) $g'(0) = 6 > 0$, $g'(2) = 4 > 0$ so $x = 1$ is <u>neither a max or a min</u>

(ii) $g'(5) = 16 > 0$, $g'(7) = -36$, so $x = 6$ is a <u>relative max</u>

c (i) $h(x) = -x^3 + 6x^2 - 48x + 150$

$h'(x) = -3x^2 + 12x - 48$

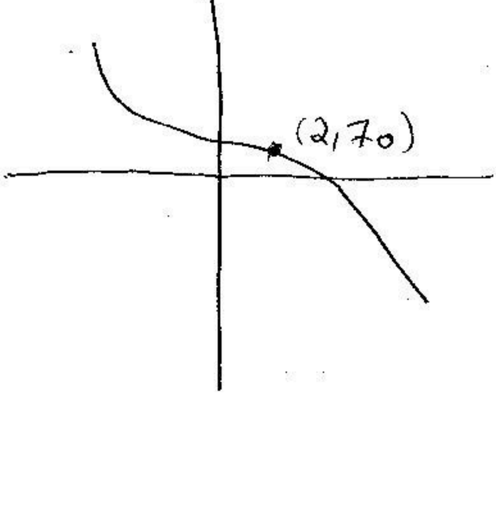
$h''(x) = -6x + 12$, Solve $h''(x) = 0$

$$-6x + 12 = 0 \Rightarrow -6x = -12, \quad x = \frac{-12}{-6} = 2$$

$h(2) = -2^3 + 6(2^2) - 48(2) + 150 = 70$

inflection point at $(2, 70)$.

(ii) $(2, 70)$ is a point of <u>slowest decrease</u>



$X_{min} = -4$

$X_{max} = 10$

$Y_{min} = -730$

$Y_{max} = 502$

many other windows will work

Solutions to Final Exam Review Exs
page 6

10. $F(x) = \int (10x^4 - \frac{15}{x})\,dx$ and $F(2) = 16$

so $F(x) = \frac{10x^5}{5} - 15\ln|x| + C$

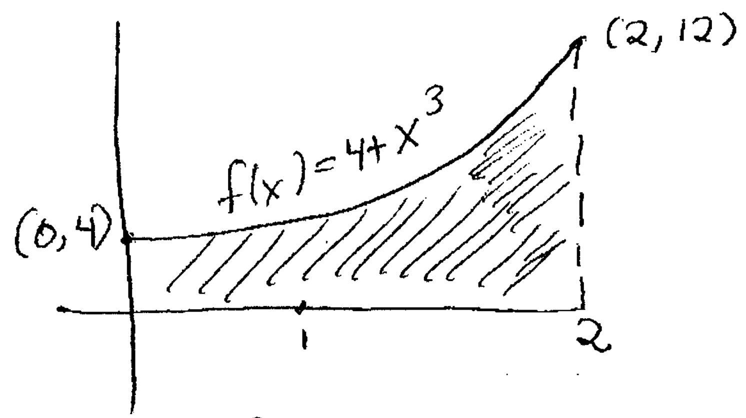
$= 2x^5 - 15\ln|x| + C$

$16 = F(2) = 64 - 15\ln(2) + C$

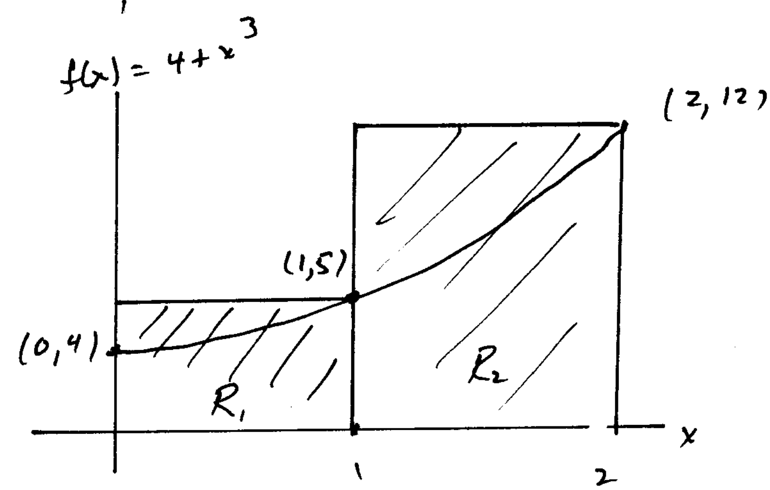
$C = 15\ln 2 - 48 \approx -37.6028$

$F(x) = 2x^5 - 15\ln|x| - 37.6028$

11a.



11b



R_1: ht = 5, base = 1
AREA = 5

R_2: ht = 12, base = 1
AREA = 12

TOTAL AREA $\approx 5 + 12 = 17$

12a. $\int_{-1}^{5} 1.75e^{0.25x}\,dx = \left[\frac{1.75}{0.25} e^{0.25x}\right]_{-1}^{5}$

$= \left[7e^{0.25x}\right]_{-1}^{5} = 7e^{1.25} - 7e^{-0.25} \approx 18.9808$

b. $\int_{1}^{4} (64x^{-5} + 3x^{-1/2})\,dx = \left[64\frac{x^{-4}}{-4} + \frac{3}{1/2}x^{1/2}\right]_{1}^{4}$

$= \left[-16x^{-4} + 6x^{1/2}\right]_{1}^{4} = \left(-\frac{16}{4^4} + 6\sqrt{4}\right) - \left(-\frac{16}{1^4} + 6\sqrt{1}\right)$

$= -\frac{1}{16} + 12 + 16 - 6 \approx 21.9375$

13a $\int_{1}^{6} e(t)\,dt = \int_{1}^{6} -0.6891t^2 + 3.287t - 3.1306\,dt$ thousand employees

$= \left[-0.6891\cdot\frac{t^3}{3} + 3.287\frac{t^2}{2} - 3.1306t\right]_{1}^{6}$

$= \left(-0.6891\cdot\frac{6^3}{3} + 3.287\cdot\frac{6^2}{2} - 3.1306\cdot 6\right)$

$- \left(-\frac{0.6891}{3} + 3.287\cdot\frac{1}{2} - 3.1306\right)$

$= (-9.2328) - (-1.7168) = -7.516$ thousand employees

or fnInt$(-.6891x^2 + 3.287x - 3.1306, x, 1, 6) = -7.516$ thousand employees

b. Between 1992 and 1997 the number of employees fell by 7,516.

14. Consumer surplus $= \int_{60}^{90} D(p)\,dp$. The unit is 100 games $\cdot \frac{\text{dollars}}{\text{game}}$

= 100 dollars.

$$\int_{60}^{90} (-0.09p^2 + 1296)\,dp = \left[-\frac{.09p^3}{3} + 1296p\right]_{60}^{90}$$

$$= \left(-\frac{.09}{3}(90)^3 + 1296(90)\right) - \left(-\frac{.09}{3}(60)^3 + 1296(60)\right)$$

$= 94770 - 71280 = 23490$ hundred dollars

$= \$2,349,000$

15. $A(t) = 10200(1.06)^t$

a) $A'(t) = 10200(1.06)^t \ln(1.06)$ \$/yr

b) $A(20) = \$32712.78$

c) Average balance $= \frac{1}{15-5}\int_5^{15} A(t)\,dt = \frac{1}{10}\int_5^{15} 10200(1.06)^t\,dt$

$$= \frac{1}{10}\left[\frac{10200(1.06)^t}{\ln(1.06)}\right]_5^{15}$$

$$= \frac{1}{10}\,\frac{10200(1.06)^{15}}{\ln(1.06)} - \frac{1}{10}\,\frac{10200(1.06)^5}{\ln(1.06)} = \$18526.16$$

d) $A'(20) = 1906.14$ dollars/year

e) interest earned from year 20 to year 21 $\approx$ 1 year $\cdot A'(20)$

$= 1906.14$

f) $A(21) - A(20) = \$1962.77$

g) first arrow from left is part (f); second arrow is part (b); third arrow is part (e); fourth arrow is part (d)

16. $f(x) = 3x^2 - 2x$

a) $AROC = \dfrac{f(x+h) - f(x)}{h} = \dfrac{\left(3(x+h)^2 - 2(x+h)\right) - \left(3x^2 - 2x\right)}{h}$

$= \left(3(x^2 + 2xh + h^2 - 2x - 2h - 3x^2 + 2x\right)/h$

$= \left(3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x\right)/h$

$= \left(6xh + 3h^2 - 2h\right)/h$

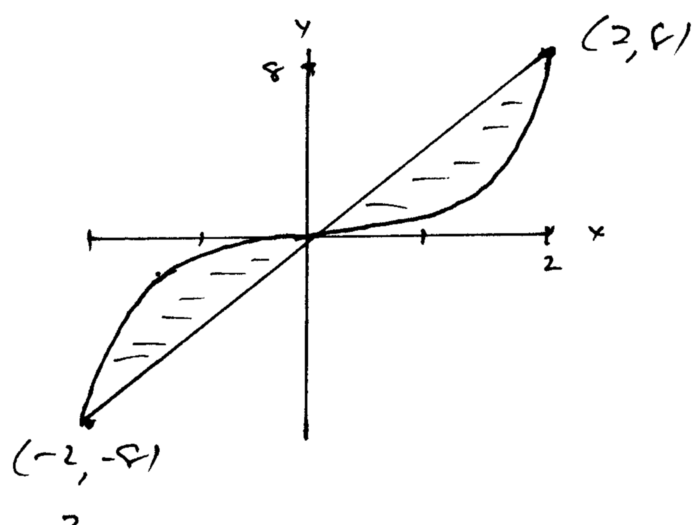
$= 6x + 3h - 2$

b) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (6x + 3h - 2)$

$f'(x) = 6x - 2$

17.

a)



b) $2\int_0^2 (4x - x^3)\,dx = 2\left(2x^2 - \frac{1}{4}x^4\right)\Big|_0^2 = 2(8-4) = 8$