MTHU131 Midterm Review

1. The profit, in millions of dollars, of a company t years after 1992 is given in the following table:

Years since 1992	0	2	4	6	8	10
Profit, in mils. of \$.	5	220	260	220	200	280

a) Let t be the number of years since 1992. Fit a model for the profit, $P(t)$. Give your model with four decimal places.

$$P(t) = 1.9387t^3 - 33.1647t^2 + 165.3009t + 5.4365$$

b) From the model in part a), derive a function, $P'(t)$, giving the rate of change of the profit t years after 1992.

$$P'(t) = 5.8161t^2 - 66.3294t + 165.3009$$

c) According to the model, how fast was the profit changing in 2002? Show work. Give your answer with units.

$P'(10) = 83.6169$ Millons of Dollars per year (using the formula in part (b))

or nDeriv $(P(x), x, 10) = 83.6045$ Millons of Dollars/yr, where $P(x)$ is The exact Model

d) According to the model, what will the profit be in 2003? Show work. Give your answer with units.

$P(11) = 391.2274$ Million Dollars (Using The formula in Part (b))

or $P(11) = 391.1731$ million Dollars, where $P(x)$ is The Exact Model.

2. The following model gives the population in thousands, of an island, t years after 1990:

$$N(t) = \frac{1020}{1 + 3e^{-0.037t}} + 200$$

a) According to the model, what was the population of the island in 2002? Show work. Give your answer with units.

$N(12) = 548.79$ Thousand People

b) Write down a function that gives the rate of change of the population t years after 1990.

$$N'(t) = -1020\,(1+3e^{-.037t})^{-2}(3e^{-.037t})(-.037)$$
$$= 113.22\,(1+3e^{-.037t})^{-2}\,e^{-.037t}$$

c) How rapidly was the population changing in 2002? Show work. Give your answer with units.

$N'(12) = 8.49$ Thousand people/yr.

Explain the meaning of your answer in a complete sentence with units. Do not use words like "rate", "per", "derivative" or "slope".

From 2002 to 2003 The population of the island increased by about 8.49 Thousand people

3. Let $f(x) = -3x^2 + 5x - 1$. Find the average rate of change of $f(x)$ between the points $(x, f(x))$ and $(x + h, f(x + h))$. Show all your algebra and **simplify your answer.**

$$AROC = \frac{f(x+h) - f(x)}{h} = \frac{(-3(x+h)^2 + 5(x+h) - 1) - (-3x^2 + 5x - 1)}{h}$$

$$= (-3x^2 - 6xh - 3h^2 + 5x + 5h - 1 + 3x^2 - 5x + 1)/h$$

$$= (-6xh - 3h^2 + 5h)/h = -6x - 3h + 5$$

4. Find the derivative of each of the functions below:

a))$g(x) = \frac{5}{x^4} + 12e^x$ $= 5x^{-4} + 12e^x$

$$g'(x) = -20x^{-5} + 12e^x$$

b) $h(x) = 6\sqrt[3]{4x^2 + 15}$ $= 6(4x^2 + 15)^{1/3}$

$$h'(x) = 6(\tfrac{1}{3})(4x^2 + 15)^{-2/3}(8x)$$

$$h'(x) = 16x(4x^2 + 15)^{-2/3}$$

c) $m(x) = 7\ln(x^7 + 5x^3) - 4x^{10}$

$$m'(x) = 7 \cdot \frac{7x^6 + 15x^2}{x^7 + 5x^3} - 40x^9$$

d) $k(x) = \frac{4(3^x)}{\sqrt{x}}$ $= 4(3^x)x^{-1/2}$

$$K'(x) = 4(3^x)(-\tfrac{1}{2}x^{-3/2}) + 4x^{-1/2} \cdot 3^x \ln 3$$

5. If \$21,500 is put into an account at 2.2% compounded monthly, give the formla for the amount in the account after t years.

$$A(t) = 21500\left(1 + \frac{.022}{12}\right)^{12t}$$

6. The amount (dollars) in a separate account (from the one in problem 4) after t years is given by: $A(t) = 32,000e^{.02t}$.

a) Write down the function giving the rate at which your account will be changing after t years.

$$A'(t) = 32000\, e^{.02t}(.02) = 640e^{.02t}$$

b) At what rate will the amount in your account be changing after 8 years? Show work and give your answer with units.

$$A'(8) = 640\, e^{.02(8)} = 751.05 \text{ dollars per year}$$

Explain what this means in a sentence.

From the end of the eighth year to the end of the ninth year, the account balance increases by roughly \$751.05

c) How much do you have in your account after 8 years? Show work and give your answer with units.

$$A(8) = \$37,552.35$$

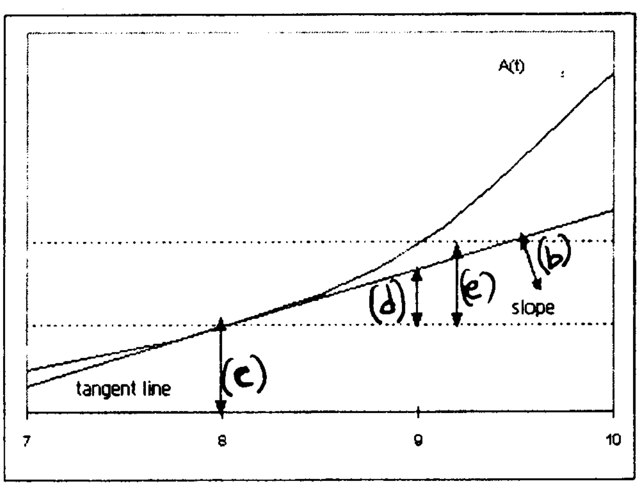
d) Using a derivative, estimate the amount of interest that you will earn from the 8th to the 9th year. Show work. Give the units.

Since $A'(8) = 751.05$, you will earn about \$751.05 in interest.

e) Compute the actual amount of interest that you earn from the 8th to the 9th year. Show work. Give your answer with units.

$$A(9) - A(8) = 32000\, e^{(.02)(9)} - 32000\, e^{(.02)8} = \$758.61$$

f) Part of the graph of $A(t)$ and its tangent line at $t = 8$ are shown below. Label the features (arrow pointers) that correspond to your answers to parts (b), (c), (d), and (e) above.



7. W inc. has found that the number of sunglasses it sells is given by the model: $D(x) = 25,000(0.965)^x$ where x is the selling price in dollars.

a) Write down a model for $R(x)$ (dollars), the revenue, as a function of the price.

$$R(x) = 25,000(.965)^x x$$

b) Write down a formula for the rate of change of revenue as a function of price.

$$R'(x) = 25000(.965)^x (x)' + (25000(.965^x))' x \quad \text{(product rule)}$$
$$= 25000(.965)^x + 25000(.965)^x \ln(.965) x$$

c) When the price is \$30, what is the rate of change of revenue? Show work, give your answer with units.

$R'(30) = -590.81$ dollars per dollar

or nDeriv $(R(x), x, 30) = -590.81$ dollars per dollar

d) If W inc. wants to maximize revenue, should it increase the price of sunglasses from \$30 to \$30.50? Explain the answer in a sentence using your answer to part c).

From part C, we know that $R'(30)$ is negative. This means if we increase the price a little beyond \$30, revenue will decrease. Therefore, the price should not be increased to \$30.50.

e) W inc. has fixed costs of \$7,500 and the sunglasses cost \$9 each to make. Write down a model for the cost function $C(x)$ (dollars).

$$C(x) = 9 \cdot D(x) + 7500 = 9(25,000)(.965)^x + 7500$$
$$= 225,000(.965)^x + 7500$$

f) Write down a formula for the profit $P(x)$ (dollars) and find the profit when the sunglasses are sold for \$30. Show work, give your answer with units.

$$P(x) = R(x) - C(x) = 25000(.965)^x x - 225,000(.965)^x - 7500$$

$P(30) = \$172792.93$

8. A company makes golf tees at a daily cost of $C(x) = 0.1x^3 - 0.5x^2 + 5.9x + 145.5$ dollars where x is the number of hundreds of golf tees produced.

a) Find the marginal cost function.

Marginal Cost function $= C'(x) = 0.3x^2 - x + 5.9$

b) Find the marginal cost of producing 500 golf tees. Show work. Give your answer with units.

$C'(5) = 0.3(5)^2 - 5 + 5.9 = 8.4$ dollars/hundred golf Tees

c) Assume that it costs \$175 to produce 500 golf tees. Use this information and your answer to part (b) to **estimate** the cost of producing 550 golf tees. Show work. Give your answer with units.

$C(5.5) \approx C(5) + C'(5)(0.5) = 175 + (8.4)(0.5) = 179.2$

\$179.20