MATH 1231 Quiz 5 Review

1. Fred sells specialty pens. He believes that the demand for the (number of) pens can be modeled by $D(x) = 50 - 4x$, where x is the selling price of a pen in dollars.

a) Write down the revenue function (dollars) in terms of x.

$$R(x) = x \cdot D(x) = x(50 - 4x) = 50x - 4x^2$$

b) If the unit cost of each pen is \$4.00, write down the profit function (dollars) in terms of x.

$$P(x) = R(x) - C(x) = (50x - 4x^2) - 4(50 - 4x)$$

$$P(x) = -4x^2 + 66x - 200$$

c) Use derivatives and algebra to find the price that maximizes profit. Show your work. Use the calculator only to do basic arithmetic.

$P'(x) = -8x + 66$, $P'(x) = 0$ for $x = 66/8 = 8.25$

\$8.25

d) Show in TWO DIFFERENT WAYS that the price you found in (c) gives the maximum profit.

$P'(x) > 0$ for $0 < x < 8.25$ and $P'(x) < 0$ for $8.25 < x < 12.5$

OR

$P''(x) = -8 < 0$ for all x $\Longrightarrow$ The graph is C. Down and the Slope is zero at $x = 8.25$ $\therefore$ Max at $x = 8.25$

e) Write a sentence, with units, that gives the price that maximizes profit, the maximum profit and the number sold.

If Fred Charges \$8.25 for a pen then his profit of \$72.25 will be a Maximum. At this price/pen he will sell 17 pens.

2. The cubic function $j(x)$ has critical points at $x = -4$ and $x = 5$. Values of $j''(x)$, the **second** derivative of $j(x)$ are given in the table below.

x	-5	-4	0	5	6
$j''(x)$	-11	-9	-1	9	11

a) Which (if any) of the critical points of $j(x)$ are relative maxima of $j(x)$? Explain your reasoning.

$x = -4$ because $j''(-4) < 0$ at this C.pt.

b) Which (if any) of the critical points of $j(x)$ are relative minima of $j(x)$? Explain your reasoning.

$x = 5$ because $j''(5) > 0$ at this C.pt.

3. If $f'(x) = 5x^2 - 3x - 8$, **USE BASIC ALGEBRA** to find the x-coordinate of each critical point of $f(x)$. Write down the equation you solve to find the critical points, and SHOW the steps to solve it. (Note: You are given the **DERIVATIVE** of $f(x)$.)

$f'(x) = (5x-8)(x+1)$; $f'(x) = 0$ for $x = 8/5$, $x = -1$

C.pts: $8/5$, -1

4. Suppose $g(x)$ has critical points at $x = 0$ and at $x = 6$. If $g'(x) = x^3 - 6x^2$, circle the correct statement for each critical point. Show work: (Note: You are given the **DERIVATIVE** of $g(x)$.) $g'(x) = x^2(x-6)$

(a) $x = 0$ is a relative max relative min neither

WORK: No Sign Change in $g'(x)$ near $x = 0$

(b) $x = 6$ is a relative max relative min neither

WORK:

$g'(x)$ Changes signs from negative to positive at $x = 6$

5. Let $h(x) = -x^3 + 9x^2 - 30x + 20$. Put this function into your calculator as Y_1 and check that $Y_1(2) = -12$.

a) **Use derivatives and basic algebra** to find the inflection point of the function $h(x)$. Give the x and y coordinates of the inflection point. Show your work especially any equations you solve and how you solve them.

$h'(x) = -3x^2 + 18x - 30$

$h''(x) = -6x + 18$

$h''(x)$ Changes sign at $x = 3$ $\Rightarrow$ inflection at $(3, h(3)) = (3, -16)$

b) Graph $h(x)$ on your calculator over the interval $-2 \le x \le 6$ and sketch what you see. In your sketch, the inflection point should be clearly labelled. Write down the window settings you use. Use the graph to decide what kind of inflection point the graph has. Circle one of the following.

Point of fastest increase Point of slowest increase

Point of fastest decrease Point of slowest decrease

Window settings

$X_{min} = -2$, $X_{max} = 6$, $Y_{min} = -30$, $Y_{max} = 30$.

GRAPH: